# CENTRAL UNIVERSITY OF TAMIL NADU, THIRUVARUR
# SCHOOL OF MATHEMATICS AND COMPUTER SCIENCES

## M.Sc. STATISTICS AND APPLIED MATHEMATICS

## COURSE STRUCTURE 2020-2021

## SAM011 ANALYSIS I 4 Credits

**Preliminaries:** Recalling Limit, Continuity and differentiation**.**

**UNIT I:** Sequences and Series of Real Numbers**:** Convergent and Divergent of Sequence, Cauchy Sequence, Upper Limit and Lower Limit of Real Sequences. Cauchy Criterion for Series of Numbers, Absolute Convergence, Series of Non-Negative Real Numbers, Geometric Series, The number e, Cauchy Product of Series, Merten's Theorem, Rearrangement of Series, Riemann's Theorem on Rearrangement of Series, Riemann-Stieltjes Integral: Definition, Existence of the Integral, Properties of the Integral, Integration and Differentiation, Rectifiable Curve**.** **(20L)**

**UNIT II:** Sequences of Functions**:** Point-wise Convergence, Uniform convergence, Uniform Convergence and Continuity, Uniform Convergence and Integration, Uniform Convergence and Differentiation, Equi-continuous Family of Functions, Stone-Weierstrass Theorem**.** **(10L)**

**UNIT III:** Measure**:** Length of Open and Closed Set, Inner and Outer Measure, Measurable Sets, Regularity, Borel and Lebesgue Measurability, Abstract Measure, Extension of a Measure, Completion of a Measure. Measurable Functions: Simple Measurable Functions, Sequence of Measurable Functions and their convergence**.**

**(15L)**

**UNIT IV:** Lebesgue Integration**:** Integrals of simple functions, Integrals of Non Negative Functions, Fatou's Lemma, Monotone Convergence Theorem, Dominated Convergence Theorem, Integration of Series, Riemann and Lebesgue Integrals, Product Measure, Fubini's Theorem**.** **(15L)**

**References:**


1. W**.** Rudin, Principles of Mathematical Analysis, 3rd Edition, McGraw-Hill, 1984**.**
2. T**.** Apostol, Mathematical Analysis, 2nd EdItion, Narosa Publishing House, 1985**.**
3. G**.** de Barra, Measure Theory and Integration, New Age International (P) Limited, 1996**.**
4. H**.** L**.** Royden, Real Analysis, 3rd Edition, McMillan Publication Co.Inc., 1988**.**
5. Billingsley P. Probability and measure. John Wiley & Sons; 2008.

## SAM012 LINEAR ALGEBRA 4 Credits

**UNIT I:** Vector Spaces; Subspaces; Linear combinations and span, Linear dependence and independence, Bases and dimension of a vector space; Vector spaces with inner products; Gram-Schmidt orthogonalization; Linear Transformations (**LT**) and its properties; Representation of transformations by matrices; Invertibility and isomorphisms; Change of bases; Orthogonal transformation; Dual spaces**. (15L)**

**UNIT II:** Systems of linear equations **-** theory and computational aspects; Homogeneous and non-homogeneous systems of linear equations; Existence and uniqueness of solutions; Matrices and elementary row operations; Row-reduced echelon matrices; Gaussian elimination method; Rank of a matrix; Inverses, G-inverse and transposes of a matrix**. (15L)**

**UNIT III:** Eigen values and Eigen vectors of an **LT**; Diagonalization of **LT**; Properties of Eigen values and Eigen vectors; Cayley-Hamilton theorem and its applications; Minimal polynomial for an **LT**; Eigen values of matrix polynomials; Matrix limits and Markov chains; Operators on complex and real vector spaces; Orthogonal projections and the spectral theorem; Similar linear transformations; Positive definite matrices and least squares. **(15L)**

**UNIT IV:** Bilinear forms; Quadratic forms; Rank, index, and signature of quadratic forms; Reduction of quadratic form into a canonical form; Decomposition of a vector space into the Jordan form; Single value decomposition and its applications; Sylvester's law for congruent matrices; Its application in Einstein's special theory of relativity; Generalized eigen value problem; Method of Lagrange multipliers**. (15L)**

**References:**


1. G. Strang, Linear Algebra and its Applications, 4th Edition, Cengage Learning India Pvt Ltd., 2005.
2. Ramachandra Rao, A. and Bhimasankaram, P. (2000). Linear Algebra. Hindustan Book Agency
3. S. H. Friedberg, A. J. Insel, and L. E. Spence, Linear Algebra, 4th Edtion, Prentice-Hall of India, 2003.
4. Searle, S. R. (1982). Matrix Algebra Useful for Statistics, John Wiley, New York
5. S. Axler, Linear Algebra Done Right (Undergraduate texts in Mathematics), Springer, 2nd Edition, 1997.
6. Lay, D. C. Lay, S. R. and Mc Donald, J. J. (2016) .Linear Algebra and Its Applications, Fifth Edition, Pearson, Boston.

## SAM013 PROBABILITY DISTRIBUTIONS 4 Credits

**UNIT I:** Introduction - Sample space and events - Axiomatic approach to probability - Conditional Probability and Independence- Law of multiplication - Law of total probability and Bayes' Theorem, Discrete and continuous random variables - probability mass function and density function - distribution function – Expectation, variance and quantiles**.** **(15L)**

**UNIT II:** Probability models for categorical, count and continuous data. Joint probability distributions - marginal and conditional distributions-Independent random variables, Conditional expectation**.** **(15L)**

**UNIT III:** Moments and moment generating functions- Sums of independent random variables - Limit theorems: Markov and Chebyshev inequalities, Law of Large numbers, convergence and distributions - Central Limit Theorem**.** **(15L)**

**UNIT IV:** Mixtures of probability distributions, decomposition of mixture type CDF into discrete and continuous CDF's, expectation and variance of mixture distributions, Transformations of univariate random variables, Transformations of bivariate random variables**.** **(15L)**

**References:**


1. V. K. Rohatgi and A. K. M. D. Ehsanes Saleh, An introduction to Probability and Statistics, 2nd Edition, Wiley Eastern Ltd., 2001.
2. S. Ross, A first Course in Probability, 6th Edition, Pearson Education, 2006.
3. Berger, R. and Casella G. (2002). Statistical Inference, Duxbury Resource Center, Second Edition.
4. Hogg, R. V. McKean, J. W. and Craig, T. T. (2005). Introduction to Mathematical Statistics, Sixth Edition, Pearson Prentice Hall, New Jersey.
5. Dasgupta, A. (2010) Fundamentals of Probability: A First Course, Springer, New York.
6. S. Ghahramani, Fundamentals of Probability with Stochastic Processes, Pearson education, 2012.
7. R. E. Walpole, Probability and Statistics for Engineers and Scientists, Prentice Hall, 2012.

## SAM014 STATISTICAL COMPUTING WITH "R" 4 Credits

**UNIT I:** Introduction to R - A programming language and environment for data analysis and graphics. Syntax of R expressions: Vectors and assignment, vector arithmetic, generating regular sequence, logical vector, character vectors, Index vectors; selecting and modifying subsets of data set, Data objects: Basic data objects, matrices, partition of matrices, arrays, lists, factors and ordered factors, creating and using these objects; Functions- Elementary functions and summary functions, applying functions to subsets of data. **(15L)**

**UNIT II:** Data frames: The benefits of data frames, creating data frames, combining data frames, Adding new classes of variables to data frames; Data frame attributes. Importing data files: import.data function, read. table function; Exporting data: export.data function, cat, write, and write. table functions; Outputting results - sink function, formatting output - options, and format functions; Exporting graphs - export. graph function. **(15L)**

**UNIT III:** Random numbers from various distributions like uniform, Normal, gamma, exponential, beta, F, poisson, binomial, etc Graphics in R: creating graphs using plot function, box plot, histogram, line plot, stem and leaf plot, pie chart, bar chart multiple plot layout, plot titles, formatting plot axes. Interactively adding information of plot - Identifying the plotted points, adding trend lines to current scatter plot, adding new data to current plot, adding text and legend. **(15L)**

**UNIT IV:** Loops and conditional statements: Control Statements; if statement, if else Statement. Looping statement; for loop, repeat, while loop Developing simple programs in R for data analysis tasks, saving programs, executing stored programs, defining a new binary operator, assignment within function, more advanced examples, object oriented programme. Creating function libraries- library function, attaching and detaching the libraries. **(15L)**

**References:**


1. J.M. Chambers, Programming with Data: A guide to S language, Springer,1998.
2. W. N. Venables and B. D. Ripley, S Programming, Springer, 2000.
3. B. S. Everitt, A handbook of Statistical Analysis using S-Plus, Chapman & Hall, 1994.
4. P. Dalgaard, Statistics and computing: Introductory Statistics with R, Springer, 2002.
5. J. Maindonald and J. Braum, Data Analysis and Graphics Using R: An example-based approach Second Edition, Cambridge Series in Statistical and Probabilistic Mathematics, 2007.

## SAM021 ANALYSIS II 4 Credits

**UNIT I: (Basic Complex Analysis)**
Complex Numbers, geometric representation, powers and roots of complex numbers. Functions of a complex variable. Analytic functions. Cauchy-Riemann equations. Elementary functions. Conformal mapping (for linear transformation), Contours and contour integration. Cauchy's theorem, Cauchy integral formula. Power Series, term by term differentiation, Taylor series, Laurent series, Zeros, singularities, poles, essential singularities, Residue theorem and its Applications. **(20L)**

**UNIT II: (Functional Analysis)**
Banach spaces, Riesz Lemma (On compactness of the unit ball in a normed linear space), Bounded linear maps on finite and infinite-dimensional normed linear spaces: Hahn Banach Theorem (geometric and extension forms), characterization of finite-dimensional normed linear spaces, Fundamental theorems on Banach spaces-Uniform Boundedness Principle, Closed Graph Theorem, Open Mapping Theorem. **(15L)**

**UNIT III:** Dual spaces of some classical spaces, Hilbert spaces: Gram-Schmidt orthonormalization process, Bessel's inequality, orthonormal basis, Riesz Representation Theorem-Dual of a Hilbert space. **(10L)**

**UNIT IV:** Bounded operators on a Hilbert space: Adjoint of an operator, orthogonal projections, self-adjoint, normal and unitary operators, Introduction to Banach Algebras-Spectrum of an operator, Spectral Theorem for finite dimensional Hilbert spaces. **(15L)**

**References:**


1. J. W. Brown and R. V. Churchill, Complex Variables and Applications, McGraw-Hill, 2004.
2. L. V. Ahlfors, Complex Analysis, McGraw-Hill, 1966.
3. B.V. Limaye, Functional Analysis, 2nd Edition, New Age International, 1996.
4. E. Kreyzig, Introduction to Functional Analysis with Applications, Wiley, 2007.
5. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
6. A. E. Taylor and D.C. Lay, Introduction to Functional Analysis, Second Edition, Wiley, 1980.

## SAM022 MULTIVARIATE STATISTICAL ANALYSIS (Theory and Lab) 4 Credits

**UNIT I:** Reviews of Multivariate Distributions, Multiple and Partial Correlation, Multivariate Normal Distribution, Marginal and Conditional Distributions - Maximum likelihood Estimators of sample Mean and dispersion Matrix. Distribution of mean vector and Sample Dispersion Matrix - James-Stein Estimator for the Mean Vector, Wishart Distribution and its Properties (without derivation) **(15L)**

**UNIT II:** Tests based on Mean Vectors for one and two Multivariate Normal Distributions -Hotelling'sT2 and MahalanobisD2 test statistics with their null and non-null distributions - Related Confidence Regions - Testing and Illustration using likelihood Ratio Criterion**.** **(15L)**

**UNIT III:** Principal Component Analysis, Factor Analysis Underlying Models and Illustrations, Identification Problem, Estimation – Maximum likelihood Method, Centroid Method, Canonical Correlation – Extraction - Properties**.** **(15L)**

**UNIT IV:** Classification Analysis using Discriminant functions - Clustering techniques Hierarchical Clustering - Agglomerative techniques, Single Linkage Method, Complete average linkage method–Non-hierarchical method–K-Mean**. (15L)**

**References:**


1. T. W. Anderson, An Introduction to Multivariate Statistical Analysis, Second Edition, Wiley Eastern, 1980.
2. R. A. Johnson and D. W. Wichern, Applied Multivariate Statistical, 5th Edition, Upper Saddle River, NJ: Prentice hall; 2002.
3. M. Jambu and M. O. Lebeaux, Cluster Analysis and Data Analysis, North Holland Publishing Company, 1983.
4. A. M. Kshirsagar, Multivariate Analysis, Marcel Decker.1972.
5. Härdle, W. K. & Simar, L. (2012). Applied Multivariate Statistical Analysis, Springer, New York
6. D. F. Morrison, Multivariate Statistical Methods, Second Edition, McGraw Hill, 1976.

## SAM023 NUMERICAL ANALYSIS (Theory and Lab) 4 Credits

**UNIT I: Linear and Nonlinear Equations:** Linear System (Direct methods); Gaussian elimination with pivoting and scaling; LU decomposition; Vector and matrix norms; Error analysis and system condition; Linear system (Iterative methods); Jacobi and Gauss-Seidel method; Convergence considerations; The Eigenvalue problems; Nonlinear equations; One-point iteration approach; Newton's method. **(20L)**

**UNIT II: Interpolation by Polynomials:** Lagrange's interpolation; Inverse interpolation; Accuracy of interpolation; Newton's divided differences interpolation; Errors in Lagrange's and Newton's divided differences interpolation; Relationship between derivatives and divided differences; Least squares approximation; Approximation by trigonometric polynomials; Fast Fourier transforms; Interpolation by splines. **(15L)**

**UNIT III: Numerical Differentiation and Integration:** Differentiation based on interpolation and divided differences; Newton-Cotes integration formulas (trapezoidal & Simpson rules); Gaussian quadrature; Error estimation in trapezoidal rule, Simpson rules, and Gaussian quadrature; Quadrature rules for multiple integrals. **(10L)**

**UNIT IV: Numerical Solution of Differential Equations: ODEs -** Single-point methods; Multipoint methods; Error estimation and convergence of the above methods; Finite difference methods; Consistency, order, stability, and convergence of numerical methods; **PDEs -** Steady-state two-dimensional (2D) Laplace equation; Finite difference solution of the Laplace equation; Unsteady 1D parabolic diffusion equation, Explicit and implicit schemes; Unsteady 1D convection hyperbolic equation; Explicit schemes for convection equation. **(15L)**

**References:**


1. G. M. M. Phillips and P. J. Taylor, Theory and Applications of Numerical Analysis, Second Edition, Elsevier, 2006.
2. E. Isaacson, H. B. Keller, Analysis of Numerical Methods, First Edition, Dover Publication, 1994.
3. A. Quarteroni, F. Saleri and P. Gervasio, Scientific Computing with MATLAB and Octave, Springer, 2006.
4. S. D. Conte and C. de Boor, Elementary Numerical Analysis: An Algorithmic Approach, Third Edition, McGraw-Hill, 1981.

## SAM024 DIFFERENTIAL EQUATIONS 4 Credits

**UNIT I:** First order linear ordinary differential equation (ODE) -The method of successive approximations, Lipchitz condition, Convergence of successive approximations, Existence and Uniqueness of solutions for first order initial value problem**.** **(15L)**

**UNIT II:** Second order linear ODE - General solution of homogeneous equations, Non-homogeneous equations, Wronskian, Method of variation of parameters, Boundary value problems, Green's functions, Sturm-Liouville problems**.** **(15L)**

**UNIT III:** First order partial differential equation (PDE), Quasi linear PDE of the first order, Integral surfaces passing through a given curve, Surfaces orthogonal to the given system, Classification of integrals, Compatible systems of first order PDE, Charpit's method, Method of Characteristics, Nonlinear partial differential equation for first order**.** **(15L)**

**UNIT IV:** Second order PDE- Origin and Classification, linear second and higher order PDE with constant and variable coefficients, Characteristics curve of the second order PDE, Canonical form, Monge's method of solution of non-linear second order PDE**.** **(15L)**

**References:**


1. M. Braun, Differential Equations and their Applications, Fourth Edition, Springer, 1993.
2. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India Ltd., 2002.
3. G. F. Simmons, Differential Equations with Applications and Historical Notes, Tata McGraw Hill, 2003.
4. F. John, Partial Differential Equations, second edition, Springer-Verlag, 1978.
5. L. C. Evans, Partial Differential Equations, AMS, 2010.

## SAM025 STATISTICAL INFERENCE 4 Credits

**UNIT I:** Sufficiency principle, factorization theorem, minimal sufficiency, minimal sufficient partition, minimal sufficient statistics, minimal sufficient statistic for exponential family, power series family, curved exponential family, and Pitman family, completeness, bounded completeness, ancillary statistics, Basu's theorem and its applications, conditionality principle. **(15L)**

**UNIT II:** Problem of point estimation, unbiased estimators, minimum variance unbiased estimator, Rao-Blackwell theorem and Lehmann-Scheffe theorem and their applications. A necessary and sufficient condition for an estimator to be UMVUE, Fisher information and information matrix, Cramer-Rao inequality. Maximum likelihood estimator (MLE), properties of MLE, MLE in nonregular families, method of scoring and its applications. **(15L)**

**UNIT III:** The concepts of prior and posterior distributions, conjugate, Jeffrey's and improper priors with examples, Bayes estimation under squared error and absolute error loss functions. **(15L)**

**UNIT IV:** Problem of testing of Hypothesis, Simple and composite hypotheses. Randomized and non-randomized tests, Most powerful test, Neyman-Pearson Lemma and its applications. Monotone likelihood ratio property, UMP test, power function of a test, existence of UMP test, UMP test for one-sided alternatives. Concept of p-value. Problem of confidence intervals, relation with testing of hypotheses problem, UMA confidence intervals, shortest length confidence intervals. Likelihood ratio test and its applications. Commonly used statistical tests. **(15L)**

**References:**


1. Rohatgi, V. K. and Saleh, A. K. MD. E. (2015). Introduction to Probability Theory and Mathematical Statistics -3rd edition, John Wiley & sons.
2. Lehmann, E. L. (1983). Theory of Point Estimation - John Wiley & sons.
3. Rao, C. R. (1973). Linear Statistical Inference and its Applications, 2nd edition, Wiley.
4. Kale, B. K. and Muralidharan, K. (2015). Parametric Inference: An Introduction, Alpha Science International Ltd.
5. Mukhopadhyay, P. (2015). Mathematical Statistics, Books and Allied (p) Ltd.
6. Dudewicz, E. J. and Mishra, S. N. (1988). Modern Mathematical Statistics, John Wiley and Sons.
7. Casella, G. and Berger, R. L. (2001). Statistical Inference, 2nd edition, Duxbury press.

## SAM031 FLUID DYNAMICS 4 Credits

**UNIT I**: Continuum hypothesis, Forces acting on a fluid; Analysis of the relative motion near a point; Transport theorem; Methods of describing fluid motion; Translation, rotation and rate of deformation; Differentiation following the motion of the fluid; Classification of fluids; Conservation laws; Equation of continuity; Euler's equation; Equations of motion (Navier-Stokes equations); Energy equation. **(15L)**

**UNIT II**: Streamlines; Equation of state (**EOS**); Isentropic fluids; Vorticity; Theory of stress and rate of strain; Relationship between them; Kelvin's circulation theorem; Helmholtz's theorem; Rotational and irrotational flows; Bernoulli's equation; Momentum theorem and its applications; Two dimensional irrotational flow of an incompressible fluid. **(15L)**

**UNIT III**: Stokes' stream function; Axisymmetric flows; Gravity waves; Damping of gravity waves; Flow in a pipe; Potential flow; Complex potential, Blasius theorem; Kutta-Joukowski theorem; D'Alembert paradox. **(15L)**

**UNIT IV**: Dimensional analysis; Law of similarity and the Reynolds number; Flow between two parallel flat plates; Couette flow; Poiseuille flow; Torque and drag on a sphere due to a uniform flow; Flow with small Reynolds numbers; Stoke's law; Unsteady motion of a flat plate; Boundary layers; Prandtl's boundary layer equations; Solution for steady flow on a flat plate of infinite length. **(15L)**

**References:**


1. G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, First Edition, 1993.
2. A. J. Chorin and J. E. Marsden, A Mathematical Introduction to Fluid Mechanics (Texts in Applied Mathematics), Third Edition, Springer-Verlag, 1993.
3. L. D. Landau and E. M. Lifshitz, Fluid Mechanics (Volume 6 of Course of Theoretical Physics), Pergamon Press, Second Edition, 1987.
4. M. E. O'Neill and F. Chorlton, Ideal and Incompressible Fluid Dynamics (Mathematics and Its Applications), Ellis Horwood Ltd, Publisher, 1986.

## SAM032 STOCHASTIC PROCESSES 4 Credits

**UNIT I:** Stochastic processes and their classification- Brownian Motion– Markov chain– Examples (Random walk, Gambler's ruin problem)- classification of states of a Markov Chain-Recurrence-Basic limit theorem of Markov Chains-Absorption probabilities and criteria for recurrence. **(20L)**

**UNIT II:** Markov chains continuous in time – General pure birth processes and Poisson process, birth and death processes, finite state continuous time Markov chains. **(15L)**

**UNIT III:** Branching processes discrete in time – Generating functions relations – Mean and variance – Extinction probabilities – Concept of Age dependent Branching process. **(15L)**

**UNIT IV:** Renewal processes – Definition and examples – key renewal theorem – Study of residual life time process. **(10L)**

**References:**


1. S. Karlin, and H. M. Taylor, A First Course in Stochastic Processes, Academic Press, 1975.
2. J. Medhi, Stochastic Processes, 3rd Edition, New age International, 2009.
3. B. R. Bhat, Stochastic Models: Analysis and Applications, New Age Publications, 2004.
4. P. W. Jones, and P. Smith, Stochastic Processes: An Introduction, Arnold Press, 2001.
5. E. Cinlar, Introduction to Stochastic Processes, Prentice-Hall Inc., 1975.
6. D. R. Cox, and H. D. Miller, Theory of Stochastic Processes, 3rd Edition, Chapman and Hall, 1983.
7. S. M. Ross, Stochastic Process, Wiley, 1983.

## List of Electives *(This list may be extended if needed)*

| Sl. No. | Course code | Course title | Credits |
|---|---|---|---|
| 1 | SAM01E | Artificial Intelligence | 4 |
| 2 | SAM02E | Biostatistics | 4 |
| 3 | SAM03E | Calculus of Variations and Integral Equations | 4 |
| 4 | SAM04E | Design and analysis of experiments | 4 |
| 5 | SAM05E | Econometrics | 4 |
| 6 | SAM06E | Advanced Topics in Differential Equations | 4 |
| 7 | SAM07E | Mathematical Modelling in Biology | 4 |
| 8 | SAM08E | Integral Transforms | 4 |
| 9 | SAM09E | Machine Learning | 4 |
| 10 | SAM10E | Mechanics | 4 |
| 11 | SAM11E | Statistical Methods in Clinical Trials | 4 |
| 12 | SAM12E | Industrial Statistics | 4 |
| 13 | SAM13E | Advanced Numerical Methods | 4 |
| 14 | SAM14E | Time Series Analysis | 4 |
| 15 | SAM15E | Introduction to Cryptography | 4 |
| 16 | SAM16E | Computational Introduction to Number Theory | 4 |
| 17 | SAM17E | Regression Analysis | 4 |
| 18 | SAM18E | Generalized Linear Models | 4 |
| 19 | SAM19E | Introduction to Fractional Calculus | 4 |

## SAM01E ARTIFICIAL INTELLIGENCE 4 Credits

**UNIT I:** The foundations of AI - The History of AI- Intelligent agents- Agent based system. Searching for solution- Uninformed/Blind search - Informed/Heuristic search - A* search - Hill-climbing search -Constraint satisfaction problem**.** **(15L)**

**UNIT II:** Logics – First order logic, Inference in first order logic, Knowledge representation. The planning problem - Planning with state space search - Partial order search - Planning with proportional logic - Planning and acting in the real world, Adversarial planning**.** **(15L)**

**UNIT III:** Uncertainty-Probabilistic reasoning - Semantics of Bayesian network - Approximate inference in Bayesian network, Exact inference in Bayesian network - Probabilistic reasoning over time**.** **(15L)**

**UNIT IV:** Learning from observation - Knowledge in learning - Statistical learning methods - Reinforcement learning**.** **(15L)**

**References:**


1. S**.** Russell and P**.** Norvig, Artificial Intelligence**:** A Modern Approach‖, Pearson Education, 2014**.**
2. D**.** Pool and A**.** Mackworth, Artificial Intelligence**:** Foundations of Computatio-al agents‖, Cambridge University, 2011**.**

## SAM02E BIOSTATISTICS 4 Credits

**UNIT I:** Functions of survival time, survival distributions and their applications viz. exponential, gamma, Weibull, Rayleigh, lognormal, death density function for a distribution having bath-tub shape hazard function. Tests of goodness of fit for survival distributions. **(15L)**

**UNIT II:** Analysis of epidemiologic and clinical data: studying association between a disease and a characteristic: (a) types of studies in epidemiology and clinical research (i) prospective study retrospective study (iii) cross-sectional data, (b) dichotomous response and dichotomous risk factor: 2x2 tables (c) expressing relationship between a risk factor and a disease (d) inference for relative risk and odds ratio for 2x2 table, sensitivity, specificity and predictivity. Cox proportional hazard model. **(15L)**

**UNIT III:** Type I and type II censoring schemes with biological examples, estimation of mean survival time and variance of the estimator for type I and type II censored data with numerical examples. Nonparametric methods for estimating survival function and variance of the estimator. Competing risk theory, indices for measurement of probability of death under competing risks and their interrelations. Estimation of probabilities of death under competing risks by maximum likelihood. Theory of independent and dependent risks. **(15L)**

**UNIT IV:** Simple and general epidemic models (by use of random variable technique). Basic biological concepts in genetics, Mendels law, Hardy-Weinberg equilibrium, random mating, distribution of allele frequency (dominant/co-dominant cases), approach to equilibrium for X-linked genes, natural selection, mutation, genetic drift, equilibrium when both natural selection and mutation are operative, detection and estimation of linkage in heredity. **(15L)**

**References:**


1. D. Collett, Modelling Survival Data in Medical Research, 3rd Edition, CRC Press, 2014.
2. L. M. Friedman, C. D. Furburg, D. L. DeMets, Reboussin and C.B. Granger, Fundamentals of Clinical Trials, 5th Edition, Springer, 2015.
3. A. Indrayan, Medical Biostatistics, 3rd Edition, CRC Press, 2012.
4. E. T. Lee, and J. W. Wang, Statistical Methods for Survival Data Analysis, Wiley, 2013.

## CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

**SAM03E** **4 Credits**

**UNIT I:** Inner Product spaces, Norm, Hilbert space, Regularity Conditions, Special kinds of Kernel, Classification of Integral Equation, Convolution Integral, Relation between Differential and Integral Equations, Classification, Conversion of Volterra Equation to Ordinary Differential Equation, Conversion of Initial Value Problem and Boundary Value Problem to Integral Equation. **(15L)**

**UNIT II:** Fredholm Integral Equations, Solution of Fredhlom Integral Equation using Decomposition Method, Direct Computation, A domain Decomposition, Successive Approximation and Successive Substitution Methods. **(15L)**

**UNIT III:** Volterra Integral Equations, Solution of Volterra Integral Equation using Successive Approximation Method, A domian Decomposition Method, Series Solution, Successive Substitution Method, Resolvent Kernel, Volterra Integral Equation of First Kind, Integral Equations with Separable Kernels. **(15L)**

**UNIT IV:** Introduction, problem of Brachistochrone, Isoperimetric problem, Variation and its properties, functions and functionals, Variational problems with the fixed boundaries, Euler's equation, Functionals in the form of integrals, special cases containing only some of the variables, Functionals involving more than one dependent variables and their first derivatives, the system of Euler's equations, Functionals depending on the higher derivatives of the dependent variables, Functionals containing several independent variables, Variational problems in parametric form. **(15L)**

**References:**


1. M. Gelfand, S. V. Fomin, Calculus of Variations, Prentice-Hall, 1963.
2. Ram P. Kanwal, Linear Integral Equations Theory and Technique, Birkhauser, 1997.
3. L. Elsgolts, Differential Equations and the Calculus of Variations, University Press of the Pacific, 2003.

## DESIGN AND ANALYSIS OF EXPERIMENTS

**SAM04E** **4 Credits**

**UNIT I:** Principles of Experimental Design: Need for designed experiments, how data are obtained, difference between data obtained from designed experiments and sampling, randomization, replication and blocking-the need for them and how to achieve these principles in an experiment**.** **(15L)**

**UNIT II:** Analysis of different designs, their strengths and when to employ which design, completely randomized design, fixed effects and random effects, randomized block design-without and with interaction**.** **(15L)**

**UNIT III:** Latin Square design, repeated LSs; incomplete block designs, balanced incomplete block design (BIBD)**.** **(15L)**

**UNIT IV:** Analysis of covariance, factorial designs-2$k$ factorial experiments, confounding, 2$k$-1 fractional factorial experiments, response surface method to determine optimum factor level combination with data obtained from a 2$k$ factorial design; split plot design; partially balanced incomplete block design (PBIBD)**.** **(15L)**

**References:**


1. D. C. Montgomery, Design and Analysis of Experiments, fifth edition, John Wiley, 2008.
2. M. N. Das, and N. C. Giri, Design and Analysis of Experiments, second edition, Wiley Eastern, 1991.
3. A. Dey, Incomplete Block Designs, Hindustan Book Agency, 2010.

## SAM05E ECONOMETRICS 4 Credits (Theory +Lab)

**UNIT I**

Introduction to Econometrics – A review of Least squares model – MLE- properties - Production and cost analysis - Theory and analysis of consumer demand specification - Estimation of demand function - Price and income elasticity of demand - Price elasticity's of supply - Torquivists model of demand for inferior goods models building bias in construction of models. **(15L)**

**UNIT II**

Introduction - Estimation and prediction – Problems of heteroscedasticity – Causes, consequences - Barlett's test- Breusch Pagan test- Goldfelf Quandt test. Autoregressive series of order 1 (AR(1)) - Lagged variables and distributed log methods - Economical Forecasting – long term and short term- Instrumental variables and Errors.

**(15L)**

**UNIT III**

Simultaneous equations model- Concept, structure and types - Identification Problem with restrictions on variance and covariance - Rank and order conditions of identifiability –Methods of estimation- Indirect least square method, two-stage least squares method of estimation and Estimation of Limited Information Maximum Likelihood (LIML)- Static and dynamic neoclassical production function- Duration models**.** **(15L)**

**UNIT IV**

K-Class estimators - Full information estimators - Full Information Maximum Likelihood
(FIML) - Three stage least squares estimators (3-SLS) and its Properties - Comparison of various estimation methods. **(15L)**

**LAB: Introduction to Economic Software-( GRETL/EViews/R/STATA/Excel)**

**References**

1. Gujarati, D.N. and Sangeetha (2007). Basic Econometrics (Third Edition). McGraw Hill Publisher, New York.
2. Castle, J. and Shephard, N. (2009). The Methodology and Practice of Econometrics. OUP Oxford publications.

3.Goldberger, A.S. (1964): Econometrics theory. John Wiley & Sons, New Delhi.

4. Kelejion, H. H. and Oates, W.E. (1988). Introduction to Econometrics, Principles and
Applications. Harper and Row Publishers Inc., New York.

5. Maddala, G.S. and Kajal Lagari (2009). Introduction to Econometrics. John Wiley & Sons,4th Edition.

6. Madnani, G.M.K. (2008): Introduction to Econometrics: Principles and Applications.
Oxford and IBH Publishing.

7. Wooldridge, J. (2012). Introduction Econometrics: A Modern Approach. Cengage Learning.

8.P. J. Dhrymes(1985), Introductory Econometrics, Springer-Verlag.

9.W. H. Greene(2006), Econometric Analysis, fifth edition, Pearson Education.

10.J. Johnston(1911), Econometric Methods, McGraw-Hill.

## ADVANCED TOPICS IN DIFERENTIAL EQUATIONS

**SAM06E** **4 Credits**

**UNIT I:** System of first order differential equations: Algebraic properties of solutions of Linear equations, Application of Linear algebra to Differential Equations, Eigenvalue and Eigen vector method, solutions of linear non homogeneous Equations. Power series solution of second order linear differential equations; regular singular points, the method of Frobenius**.** **(20L)**

**UNIT II:** Laplace Equation: Boundary value problems, Maximum and Minimum Principles for Laplace Equation, uniqueness and continuity theorems, Dirichlet problem for a circle, Neumann problem for a circle, Theory of Green's function for Laplace's equation**.** **(15L)**

**Unit III:** One Dimensional Wave equation: vibrations of a finite string, vibrations of an infinite string, vibration of a semi-infinite string, D'Alembert's solution, Separable method, existence and uniqueness of solution, Riemann method. Duhamel's principle for wave equations**.** **(15L)**

**UNIT IV:** Heat Conduction Problem in a finite rod, Heat Conduction problem for an infinite rod, Separable method, Existence and Uniqueness of the solution. Duhamel's principle for heat equations. Hadamard's definition of wellposedness**.** **(10L)**

**References:**


1. M**.** Braun, Differential Equations and their Applications, Fourth Edition, Springer, 1993**.**
2. E**.** A**.** Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India Ltd., 2002**.**
3. F**.** John, Partial Differential Equations, second edition, Springer-Verlag, 1978**.**
4. L**.** C**.** Evans, Partial Differential Equations, AMS, 2010**.**

## MATHEMATICAL MODELING IN BIOLOGY

**SAM07E** **4 Credits**

**UNIT I:** Introduction, motivation and background biology; Continuous growth models; Insect outbreak model-Spruce Budworm; Delay models; Tumour cell growth; Model with spatial Heterogeneity; Tumour cell spreading dynamics in vitro-parameter estimation**.** **(10L)**

**UNIIT II:** Models with age distribution; Logistic-type model **-** Chaos; Discrete delay models: Fishery management model; Predator-Prey models, Analysis of predator-prey model with limit cycle periodic behavior; Threshold phenomena; Lotka-Volterra systems**.** **(15L)**

**UNIT III:** Competition models; Mutualism or Symbiosis, Waves in biology; Spiral waves; Traveling wave trains in reaction diffusion systems; Spiral wave solutions of $\boldsymbol{\lambda - \omega}$ reaction diffusion systems; Enzyme kinetics; Law of mass action; Basic model for the dynamics of nerve membranes; Infectious Diseases; Simple epidemic models (SI, SIR, SEIR, and HIV), Classical Kermack-McKendrick model**.** **(20L)**

**UNIT IV:** Biological oscillators; Singular perturbation analysis for biological applications; Analysis of the phase shift equation and application to the coupled Belousov-Zhabotinskii reactions, Reaction diffusion equations; Models for animal dispersal; Pattern formation in biological systems; Cell-chemotaxis model**.** **(15L)**

**References:**


1. J. D. Murray, Mathematical Biology I**:** An Introduction (Interdisciplinary Applied Mathematics), Third Edition, Springer-Verlag Berlin Heidelberg, New York, 2002**.**
2. J. D. Murray, Mathematical Biology II**:** Spatial Models and Biomedical Applications, (Interdisciplinary Applied Mathematics), Third Edition, Springer-Verlag Berlin Heidelberg, New York, 2003**.**
3. L. Edelstein-Keshet, Mathematical Models in Biology (Classics in Applied Mathematics), Society for Industrial and Applied Mathematics (SIAM), Philadelphia, New York, 2005**.**

## SAM08E INTEGRAL TRANSFORMS 4 Credits

**UNIT I:** The Fourier Transform: Algebraic Properties, Convolution, Translation, Modulation. Analytical Properties of Fourier transforms, Transform of Derivatives and Derivatives of Transform, Perseval Formula, Inversion theorem, Plancherel's theorem, Application to solving Ordinary and Partial Differential Equations**. (15L)**

**UNIT II:** The Laplace transform: Algebraic Properties of Laplace Transform, Transform of Derivatives and Derivatives of Transform. The Inversion Theorem, Evaluation of inverse transforms by residue. Asymptotic expansion of inverse transform, Application to solving PDE and Integral Equation**. (15L)**

**UNIT III:** The Hankel transform: Elementary properties, Inversion theorem, Transform of derivatives of functions, Perseval relation, Relation between Fourier and Hankel transform, The Mellin transform: Properties and Evaluation of Transforms, Convolution Theorem for Mellin Transforms. Solving PDE by Hankel transform and Solving integral equations by Mellin Transform**. (20L)**

**UNIT IV:** The Z-Transform: Properties of the region convergence of the Z-transform. Inverse Z-Transform for Discrete-Time Systems and Signals, Signal Processing and Linear System**. (10L)**

**References:**


1. D. Loknath, Integral Transforms and their Application, CRC Press, 1995.
2. R. S. Pathak, Integral Transform for Generalized Functions and their Applications. Gordon and Breach Science Publishers, 1997.
3. F. C. Titchmash, Introduction to the theory of Fourier Integrals, Oxford Press, 1937.
4. E. J. Watson, Laplace Transforms and Application, Van Nostland Reinhold Co. Ltd., 1981.
5. E. I. Jury, Theory and Application of Z-Transform, John Wiley and Sons, 1973.
6. W. Rudin, Real and Complex Analysis, Mc. Graw Hill Inc., 1987.

## SAM09E MACHINE LEARNING 4 Credits

**UNIT I:** Introduction to Machine Learning, Different Forms of Learning, Linear Regression, Ridge Regression, Lasso, Bayesian Regression, Regression with Basis Functions. **(15L)**

**UNIT II:** Instance-Based Classification, Linear Discriminant Analysis, Logistic Regression, Large Margin Classification, Kernel Methods, Support Vector Machines, Multi-class Classification, Classification and Regression Trees. **(15L)**

**UNIT III:** Neural Networks: Multi-layer Networks, Back-propagation, Multi-class Discrimination, Training Procedures, Localized Network Structure, Deep Learning. Graphical Models: Hidden Markov Models, Bayesian Networks, Markov Random Fields, Conditional Random Fields. **(15L)**

**UNIT IV:** Ensemble Methods: Boosting - Adaboost, Gradient Boosting, Bagging - Simple Methods, Random Forest, Clustering: K-Medoids, CLARA, DENCLUE, DBCSAN. **(15L)**

**References:**


1. C. Bishop, Pattern Recognition and Machine Learning, Springer - Verlag, 2006.
2. T. Mitchell, Machine Learning, McGraw Hill Education, 2017.
3. R. O. Duda, P. E. Hart and D. G. Stork, Pattern Classification, Wiley-Black-well; 2nd edition, 2000.
4. T. Hastie, R. Tibshirani and J. Friedman, Elements of Statistical Learning, 2nd Edition, Springer, 2009.
5. Han, Jiawei, Jian Pei, and Micheline Kamber. Data mining: concepts and techniques. Elsevier, 2011.

## SAM10E MECHANICS 4 Credits

**UNIT I:** Mechanics of a particle; Mechanics of a system of particles; Generalized coordinates; Constraints; Virtual displacement; Virtual work; Principle of virtual work; D'Alembert's principle; Velocity-dependent potentials and the dissipation function; Energy and momentum. **(15L)**

**UNIT II:** Derivation of Lagrange's equations and its applications; Integrals of the motion; Routhian function; Energy function and the conservation of energy; Small oscillations for conservative mechanical system. **(15L)**

**UNIT III:** Hamilton's principle; Hamilton's equations; Derivation of Lagrange's equations from Hamilton's equation; Other variational principles; Phase space; Liouville's theorem; Hamilton principle function; Hamilton-Jacobi equation; Hamilton-Jacobi method; Separability. **(15L)**

**UNIT IV:** Differential forms and generating functions; Canonical transformations; Examples of Canonical transformations; Harmonic oscillator; Symmetric approach to canonical transformations; Special transformations; Lagrange and Poisson Brackets; Symmetry groups of mechanical systems; More general transformations. **(15L)**

**References:**


1. D. T. Greenwood, Classical Dynamics, Prentice-Hall international series in dynamics, Dover Publications, Inc., 1977.
2. H. Goldstein, Classical Mechanics, Second Edition, Narosa Publishing House, 2001.
3. J. L. Synge and B. A. Griffith, Principles of Mechanics, Third Edition, McGraw-Hill Book Company, Inc., 1970.

## STATISTICAL METHODS IN CLINICAL TRIALS

**SAM11E** **4 Credits**

**UNIT I:** Introduction to clinical trials: the need and ethics of clinical trials, bias and random error in clinical studies, conduct of clinical trials, overview of Phase I-IV trials, Multi-center trials. **(15L)**

**UNIT II:** Data management: data definitions, case report forms, database design, data collection systems for good clinical practice, concept of blinding/masking in clinical trials. Bioavailability, pharmacokinetics and pharmaco-dynamics, two compartment model. **(15L)**

**UNIT III:** Design of clinical trials: parallel vs. cross-over designs, cross-sectional vs. Longitudinal designs, review of factorial designs, objectives and endpoints of clinical trials, design of Phase I trials, design of single-stage and multi-stage Phase II trials, design and monitoring of Phase III trials with sequential stopping. **(15L)**

**UNIT IV:** Design of bio-equivalence trials, Inference for 2x2 crossover design: Classical methods of interval hypothesis testing for bioequivalence, Bayesian methods, nonparametric methods. Reporting and analysis: analysis of categorical outcomes from Phase I - III trials, analysis of survival data from clinical trials. **(15L)**

**References:**


1. C. Jennison and B. W. Turnbull (1999): Group Sequential Methods with Applications to Clinical Trials, CRC Press.
2. Chow S. C. and Liu J. P. (2004). Design and Analysis of Clinical Trials. 2nd Ed. Marcel Dekkar.
3. Chow S. C. and Liu J. P. (2009). Design and Analysis of Bioavailability and bioequivalence. 3rd Ed. CRC Press.
4. Clayton, D. and Hills, M. (2013). Statistical methods in Epidemiology, OUP.
5. Daniel, W. W. and Cross, C. L. (2012). Biostatistics: A Foundation for Analysis in the Health Sciences, 10th Edition, Wiley.
6. J. L. Fleiss (1989). The Design and Analysis of Clinical Experiments.Wiley and Sons.
7. L. M. Friedman, C. Furburg, D. L. Demets (1998). Fundamentals of Clinical Trials, Springer Verlag.
8. Marubeni. E and Valsecchi M. G. (1994). Analyzing Survival Data from Clinical Trials and Observational Studies, Wiley.
9. S. Piantadosi (1997). Clinical Trials: A Methodologic Perspective, Wiley and Sons.

## SAM12E INDUSTRIAL STATISTICS 4 Credits

**Unit I:** Shewhart Control Charts for X, R, np, p, c etc., and their uses, OC and ARL of Control Charts, Control Charts based on C.V., Modified Control Charts, CUSUM procedures, use of V-mask, Derivation of ARL. **(15L)**

**Unit II:** Decision Interval Schemes for CUSUM charts. Economic Designs of Control Charts, Pre-control, Relative Precision and Process Capability analysis and Gauge capability analysis, Multivariate Control charts and HotellingT2. **(15L)**

**Unit III:** Basic Concepts of Acceptance Sampling, Single, Double, Multiple and Sequential Sampling Plans for Attributes, Curtailed and Semi Curtailed Sampling. Dodge-Romig Tables-LTPD and AOQL protection (Single Sampling Plan only). MIL-STD-105D. **(15L)**

**Unit IV:** Variable Sampling: Assumptions, Single and Double Variable Sampling Plans. Application of Normal and Non-central t-Distributions in Variable Sampling. Continuous Sampling Plans: CSP-1, CSP-2 and CSP-3. Special Purpose Plans: Chain Sampling Plans, Skip-lot Plans. **(15L)**

**References:**


1. D. C. Montgomary, Introduction to Quality Control, John Wiley, 1985.
2. E. G. Schilling, Acceptance Sampling in Quality Control, Marcel Dekker,1982.
3. I. W. Burr, Statistical Quality Control Methods, Marcel Dekker, 1976.
4. H. J. Mittag and H. Rinne, Statistical Methods of Quality Assurance, Germany Chapman & Hall India, 1993.
5. G.B. Whetherill, Sampling Inspection and Quality Control, Halsted Press,1977.
6. E. R. Ott, Process Quality Control, McGraw Hill, 1975.
7. S. Halpern, An Introduction to Quality Control and Reliability, Prentice Hall of India, 1979.

## SAM13E ADVANCED NUMERICAL METHODS 4 Credits

**UNIT I:** Muller's and Bairstow's methods for complex equations; Successive-Over-Relaxation (**SOR**) method; Nonlinear eigenvalue problem; Householder method for non-symmetrical matrices; Approximation of Chebyshev polynomials and Chebyshev series; Pade approximation; Minimax approximations**.** **(15L)**

**UNIT II:** Minimizing a function of several variables; Bezier and B-splines curves; Interpolating on a surface; Conjugate gradient method; Linear and nonlinear programming; Some optimization problems - Transportation and stochastic problems; Weak formulation of boundary value problems (**BVPs**); Ritz-Galerkin approximation; Error estimates; Piecewise polynomial spaces; Relationship to difference methods**.** **(15L)**

**UNIT III:** Review of Sobolev and Hilbert spaces; Weak derivatives; Sobolev norms and associated spaces; Riesz representation theorem, Symmetric and non-symmetric variational formulation of elliptic and parabolic **BVPs**, Lax-Milgram Theorem; Higher order boundary value ordinary and partial differential equations - Equilibrium and propagation methods; Problems for Laplace, Poisson, diffusion, and wave equations in multidimensional space**.** **(15L)**

**UNIT IV:** Finite-Element Analysis (**FEA**) - Domain discretization; Finite element application in one and two dimensions; Rayleigh-Ritz, Collocation, and Galerkin methods; The finite element; Triangular finite elements; Rectangular elements; The Interpolant; Averaged Taylor polynomials; Error representation; Bounds for the Interpolation error**.** **(15L)**

**References:**


1. J. D. Hoffman, Numerical Methods for Engineers and Scientists, Second Edition, CRC Press (Taylor and Francis Group), 2001.
2. S. C. Brenner and L. R. Scott, The Mathematical Theory of Finite Element Methods, Springer-Verlag, 1994.
3. S. C. Chapra and R. P. Canale, Numerical Methods for Engineers, Applied Mathematics Series, Second Edition, McGraw-Hill, Inc., 1989.
4. C. Johnson, Numerical Solution of Partial Differential Equations by the Finite Element Method, Cambridge University Press, 1987.

## SAM14E TIME SERIES ANALYSIS 4 Credits

**UNIT I:** Exploratory Time Series Analysis**:** Forecasting trend and seasonality based on smoothing. Methods of Exponential and moving average smoothing, Holt-Winter method; Types and implications of interventions; Outliers, additive and innovational outliers, procedure for detecting outliers**.** **(15L)**

**UNIT II:** Stationary Stochastic models**:** weak and strong stationarity, Deseasonalising and detrending an observed time series, Auto-covariance, autocorrelation function (ACF), partial autocorrelation function (PACF) and their properties, Conditions for stationary and invertibility**.** **(15L)**

**UNIT III:** Models for Time Series**:** Time series data, Trend, seasonality, cycles and residuals, Stationary, White noise processes, Autoregressive (AR), Moving Average (MA), Autoregressive and Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) processes, Choice of AR and MA periods. Introduction to Seasonal ARIMA model, Volatility modelling**.** **(15L)**

**UNIT IV:** Spectral analysis and decomposition**:** Spectral analysis of weakly stationary process, Periodogram and correlogram analysis, Spectral decomposition of weakly AR process and representation as a one-sided MA process – necessary and sufficient conditions, implication in prediction problems**.** **(15L)**

**References:**


1. N. T. Thomopoulos, Applied Forecasting Methods, Prentice Hall, 1980.
2. G. E. P. Box, G. M. Jenkins and G. C. Reinsel, Time Series Analysis– Forecasting and Control, Pearson Education, 2004.
3. P. J. Brockwell and R. A. Davis, Introduction to Time Series and Forecasting, Springer, 2002.
4. D. C. Montgomery and L. A. Johnson, Forecasting and Time Series analysis, McGraw Hill, 1977.

## SAM15E INTRODUCTION TO CRYPTOGRAPHY 4 Credits

**UNIT I: Some topics in Elementary Number Theory:** Time estimates for doing arithmetic; Divisibility and the Euclidean algorithm; Congruences; Some applications to factoring; **Finite Fields and Quadratic Residues:** Finite fields; Quadratic residues and reciprocity**.** **(20L)**

**UNIT II: Cryptography:** Some simple cryptosystems; Enciphering matrices**.** **(10L)**

**UNIT III: Public Key:** The idea of public key cryptography; RSA; Discrete log; Kanpsack; Zero-knowledge protocols and oblivious transfer**.** **(15L)**

**UNIT IV: Primality and Factoring:** Pseudoprimes; The rho method; Fermat factorization and factor bases; The continued fraction method; The quadratic sieve method**.** **(15L)**

**References:**


1. Neal Koblitz, A Course in Number Theory and Cryptography, Second Edition, Springer-Verlag, New York, 1994**.**
2. Nigel Smart, Cryptography**:** An Introduction (Third Edition online), https://homes.esat.kuleuven.be/~nsmart/Crypto_Book/
3. J. S. Kraft and L. C. Washington, An Introduction to Number Theory with Cryptography, Chapman and Hall/CRC Press, 2013**.**

## A COMPUTATIONAL INTRODUCTION TO NUMBER THEORY

**SAM16E** **4 Credits**

**UNIT I:** Basic properties of the integers-Divisibility and primality, Ideals and greatest common divisors, Some consequences of unique factorization; Congruences - Definittions and basic properties, Solving linear congruences, Residue classes, Euler's phi function, Fermat's little theorem, Arithmetic functions and Mobius inversion**. (15L)**

**UNIT II:** Computing with large integers -Asymptotic notation, Machine models and complexity theory, Basic integer arithmetic, Computing in Z_n, Faster integer arithmetic. Euclid's algorithm - The basic and extended Euclidean algorithms, Computing modular inverses and Chinese remaindering, Speeding up algorithms via modular computation, Rational reconstruction and applications**. (15L)**

**UNIT III:** The distribution of primes - Chebyshev's theorem on the density of primes, Bertrand's postulate, Mertens' theorem, The sieve of Eratosthenes, The prime number theorem. Quadratic residues and quadratic reciprocity; Quadratic residues, The Legendre symbol, The Jacobi symbol. Computational problems related to quadratic residues; Computing the Jacobi symbol, Testing quadratic residuosity, Computing modular square roots, The quadratic residuosity assumption**. (15L)**

**UNIT IV:** Finite and discrete probability distributions - Finite probability distributeons**:** basic definitions, Conditional probability and independence, Random variables, Expectation and variance, Some useful bounds, The birthday paradox, Hash functions, Statistical distance, Discrete probability distributions**. (15L)**

**References:**


1. Victor Shoup, A computational introduction to number theory and algebra Cambridge University Press, Cambridge, 2005**.**
2. T**.** M**.** Apostol, Introduction to Analytic Number Theory, Springer-Verlag, New York, 1976**.**
3. D**.** Bressoud and S**.** Wagon, A course in Computational Number Theory, Key College Publishing, Emeryville, CA; in cooperation with Springer-Verlag, New York, 2000**.**

## SAM17E REGRESSION ANALYSIS 4 Credits

**UNIT I:** Multiple regression model, Least squares estimate (LSE), Properties of LSE, Hypothesis testing, confidence and prediction intervals, General linear hypothesis testing. Dummy variables and their use in regression analysis. Residuals and their properties, residual diagnostics. Transformation of Variables**:** VST and Box-Cox power transformation**.** **(15L)**

**UNIT II:** Variable Selection Procedures**:** R-square, adjusted R-square, Mallows' Cp, forward, backward and stepwise selection methods, AIC, BIC. Multicollinearity**:** Consequences, detection and remedies, ridge regression. Autocorrelation: sources, consequences, detection (Durbin-Watson test) and remedies. Parameter estimation using Cochrane-Orcutt method**.** **(15L)**

**UNIT III:** Nonlinear regression models**:** Nonlinear least squares, Transformation to a linear model, Parameter estimation in a nonlinear system, Statistical inference in nonlinear regression. Polynomial regression model, piecewise polynomial fitting, nonparametric regression: kernel and locally weighted regression**.** **(15L)**

**UNIT IV:** Robust Regression**:** Influential observations, leverage, outliers, methods of detection of outliers and influential observations, estimation in the presence of outliers: M-estimator, Huber loss function, breakdown point, influence function, efficiency, Asymptotic distribution of M-estimator (Statement only), Mallows' class of estimators**.** **(15L)**

**References:**


1. Draper N**.** R**.** and Smith**,** H**.** (1998)**:** Applied Regression Analysis. 3rd ed Wiley
2. Wiesberg, S**.** (1985)**:** Applied Linear Regression, Wiley**.**
3. Kutner**,** Neter**,** Nachtsheim and Wasserman (2003)**:** Applied Linear Regression Models, 4th Edition, McGraw-Hill**.**
4. Montgomery, D**.** C**.,** Peck, E**.** A**.** and Vining, G**.** (2012)**:** Introduction to Linear Regression Analysis, 5th Ed. Wiley**.**
5. Cook R**.** D**.** and Wiesberg S**.** (1982)**:** Residuals and Influence in Regression. Chapman and Hall**.**
6. Birkes, D**.** and Dodge, Y**.** (1993). Alternative methods of regression, John Wiley and Sons**.**
7. Huber, P**.** J**.** and Ronchetti, E**.** M**.** (2011) Robust Statistics, Wiley, 2nd Edition**.**
8. Seber, G**.** A**.,** Wild, C**.** J**.** (2003). Nonlinear Regression, Wiley**.**

## SAM18E GENERALIZED LINEAR MODELS 4 Credits

**UNIT I:** Generalized linear models**:** concept of generalized linear model, Link function, ML estimation, Quasi-likelihood estimation, large sample tests about parameters, goodness of fit, analysis of deviance. Residual analysis, types of residuals**:** raw, Pearson, deviance, Anscombe, quantile; residual plots. Variable selection**:** AIC and BIC**.** **(15L)**

**UNIT II:** Logistic regression**:** logit, probit and cloglog model for dichotomous data with single and multiple explanatory variables, ML estimation, large sample tests about parameters. Hosmer-Lemeshow test, ROC curve. Multilevel logistic regression, Logistic regression for Nominal response: Baseline Category model and ordinal response: Proportional odds model**.** **(15L)**

**UNIT III:** Poisson regression**:** ML and Quasi-likelihood estimation of parameters, testing significance of coefficients, goodness of fit, power family of link functions, over dispersion**:** Types, causes and remedies. Negative Binomial regression**:** NB-2 model**.** **(15L)**

**UNIT IV:** Generalized linear mixed models (GLMM)**:** Structure of the model, consequences of having random effects, estimation by maximum likelihood, marginal versus conditional models, estimation by generalized estimating equations and conditional likelihood, tests of hypothesis**:** LRT, asymptotic variance, Wald and score test**.** **(15L)**

**References:**


1. Hosmer D**.** W**.** and Lemeshow S**.** (2000)**:** Applied Logistic regression, 2nd ED. Wiley New York**.**
2. Agresti A**.** (1990)**:** Categorical Data Analysis. Wiley, New York**.**
3. R**.** Christensen (1997) Log-Linear Models and Logistic Regression, Springer. New York**.**
4. Hilbe, J**.** (2011)**:** Negative Binomial regression, Cambridge University, Press, 2nd Edition**.**
5. McCulloch, C**.** E**.,** & Searle**,** S**.** R**.** (2003). Generalized, linear, and mixed models, Wiley series in probability and statistics, New York**.**

## INTRODUCTION TO FRACTIONAL CALCULUS

**SAM19E** **4 Credits**

**UNIT I:** Fractional Calculus**:** History and Motivation, Special functions in Fractional Calculus**:** Gamma Function, Beta function, Mittag Leffler (ML) Function, Properties of ML function, Laplace Transform of Mittag-Leffler Function. Fractional Derivatives and Integrals**:** Grunwald Letnikov Fractional Derivative, Riemann Liouville (RL) Fractional Derivative, Caputo Fractional Derivative and Riemann Liouville Fractional Integral**.** **(15L)**

**UNIT II**: Analysis on Fractional Operators**:** Properties of RL Fractional Integral and Derivative, Caputo Fractional Derivative (Convergence, Boundedness), Relation in between RL and Caputo Fractional Derivative, Laplace Transform of Fractional Operators**.** **(15L)**

**UNIT III:** Linear Fractional Differential Equations (FDE)**:** Existence and Uniqueness Theorem, Successive approximation, Laplace Transform method for FDE in Homogeneous and Non homogeneous case**.** **(15L)**

**UNIT IV:** Applications of FDE**:** Physical models, Biological models, Fractional Inverse Problems. Numerical Analysis**:** ML function, Approximation of Fractional operators, Numerical solution of FDE**.** **(15L)**

**References:**


1. K. Deithelm, The Analysis of Fractional Differential Equations, Springer, Berlin, 2010.
2. K. S. Miller and B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley, New York, 1993.
3. K. B. Oldham and J. Spanier, The Fractional Calculus, Academic Press, New York, 1974.
4. I. Podlubny, Fractional Differential Equations, Academic Press, New York,1999.
5. S. G. Samko, A. A. Kilbas and O. I. Marichev, Fractional Integrals and Derivatives (Theory and Applications), Gordon and Breach Science Publishers, Amsterdam, 1993.