CENTRAL UNIVERSITY OF TAMIL NADU, THIRUVARUR

School of Mathematics and Computer Sciences

Department of Statistics and Applied Mathematics

Ph.D. APPLIED MATHEMATICS (COURSE WORK)

COURSE STRUCTURE (2020-2021)

Course code	Course title	Credits	Page No.
PSAM01	Research Methodology	4	2
PHDAM01	Principles of Mathematics	4	3
PHDAME#	Elective*	4	5-8
CPE-RPE	Research and Publication Ethics**	2	-
Total credits	**14**		

* *To be decided by DRC as per student's requirement.*

** **Note:** *The detail syllabus for this course is mentioned in UGC Letter No. D.O.No.F.1-1/2018 (Journal/CARE) dated December, 2019.*

List of Electives *(This list may be extended if needed)*

Sr. No.	Course code	Course title	Credits
1.	PHDAME1	Computational Fluid Mechanics	4
2.	PHDAME2	Advanced Theory of Differential Equations	4
3.	PHDAME3	Numerical Methods for Differential Equations	4
4.	PHDAME4	Sobolev spaces and Partial Differential Equation	4
5.	PHDAME5	Advanced Topics in Fractional Differential Equations	4

Note: A student may be allowed to earn extra credits by taking courses mentioned in the list of electives of Ph.D. Applied Mathematics and Ph.D. Statistics Course work.

Research Methodology

Course Code**:** PSAM01 Credits**:**4

Unit I: Introduction to Research Methodology: Definition, Characteristics, Objectives, Research and Scientific method, Literature review, types of research, process of scientific research, Research design. Criteria of Good Research, Defining Research Problem and Problem Solving Technique. (15L)

UNIT II: Solving Research Problems: concepts, constructs, definition, research questions, objectives and methodologies - Theoretical and Operational. Intellectual property rights (IPR) and Plagiarism; publications/patents/startups and its e-filing procedures, norms and policies. (15L)

Unit III: Art of writing: Research paper, Structure and component of research paper, research report, survey article, thesis - Presenting the research paper/thesis. Scientific Tools: Journal impact factor, Citation index, References and bibliography, Journal Communication, publishing a paper, writing a research proposal. (15L)

Unit IV: Technical Writing Tool: Introduction to LaTeX, PSTricks, Beamer, HTML and MathJaX**.** (15L)

References:

1. C. R. Kothari, Research Methodology (Methods and Techniques), New Age International Publishers, 2004**.**
2. R. Pratap, Getting Started with MATLAB (A Quick Introduction for Scientists and Engineers), Oxford University Press, Indian Edition, 2010**.**
3. N. J. Higham, Handbook of Writing for the Mathematical Sciences, 2ndEdition, SIAM, 1998**.**
4. D. E. Knuth, T. L. Larrabee, and P. M. Roberts, Mathematical Writing, Mathematical Association of America, Washington, D.C., 1989**.**
5. L. Lamport, LaTeX: A Document Preparation System, 2ndEdition (Addison-Wesley Series on Tools and Techniques for Computer T), 1994**.**
6. J. W. Best and J. V. Kahn, Research in Education, Pearson Edition, 2006**.**

Principles of Mathematics

Course Code: PHDAM01 Credits: 4

UNIT I: Review of finite-dimensional vector spaces and elementary matrix theory; Linear transformations, Change of basis, Eigen spaces; Matrix representation of linear operators and diagonalization; Diagonalization by similarity transformation, Spectral theorem for real symmetric matrices, Application to quadratic forms and difference equations, Iterative techniques, Eigenvalue problems. **(15L)**

UNIT II: The Jordan canonical form, Least squares method, Fourier series and eigenfunction expansions; Green's functions, Lie groups; Lie algebras: The Lie algebra of a matrix Lie group, Examples, Lie group and Lie algebra homomorphism, The exponential map; Basic representation theory. **(15L)**

Unit III: Functional Spaces: Normed linear spaces, Banach spaces, Quotient spaces, Finite dimensional normed spaces, Riesz lemma, Separability, Hilbert spaces, orthonormal basis, Bessel's inequality, Parseval's inequality, Dual spaces. **(15L)**

Unit IV: Operator Theory: Operators on normed linear spaces (linear, bounded and closed operators), Operators on Hilbert spaces, compact operators, singular value representation of compact operators. **(15L)**

References:

1. B. Hall, Lie Groups, Lie Algebras, and Representations: An Elementary Introduction, 2nd Edition, Springer International Publishing Switzerland, Vol. 222, 2015.
2. M.T. Nair, Functional Analysis, PHI-Learning,Fourth Print, 2014.
3. G. Strang, Linear Algebra and its Applications, 4th Edition, Cengage Learning India Pvt Ltd., 2005.
4. H. Anton, Elementary Linear Algebra with Applications, 8th Edition, John Wiley, 1995.
5. Kreyszig, Introductory Functional Analysis with Applications, Wiley, 1989.
6. S. Kesavan, Topics in Functional Analysis and Applications, Wiley Eastern, 1989.
7. B. V. Limaye, Functional Analysis, Wiley Eastern, 1981.

Computational Fluid Mechanics

Course Code: PHDAM1E Credits: 4

UNIT I: Solids, liquids and gases; Continuum mechanics; Volume forces and surface forces acting on a fluid; Mechanical equilibrium of a fluid element; Fluid properties; Ideal fluids; Viscous incompressible fluids; Gasdynamics-Theory of compressible flow: Thermodynamic aspects of gases, Important properties of compressible flows, Fundamental equations for compressible flow, Wave propagation in a compressible media; Rarefied gas dynamics; Magneto-fluid-mechanics. **(15L)**

UNIT II: The governing equations of a fluid dynamics: Their derivation, A discussion of their physical meaning; Constant density solution; Variable density solution; Forces on submerged surfaces: Forces on plane and curved surfaces; Hydrostatic lift-Buoyancy; Relative motion of liquids; Kinematics of Fluids: Methods of describing fluid motion; Translation, rotation and rate of deformation; Streamlines, path lines and streak lines; The material derivative and acceleration; Vorticity:Vorticity in polar and orthogonal curvilinear coordinates. **(15L)**

UNIT III: Mach numbers; Compressible regimes; Isentropic and characteristics states; Mach waves, Shock waves, Normal shock waves; Entropy across a normal shock; Introduction to hypersonic flow; Characteristics features of hypersonic flow; Inviscid hypersonic flow relations, hypersonic shock relations; Newtonian theory for hypersonic flows; Dimensional analysis and Similarity: The principle of dimensional homogeneity, The Pi theorem, Nondimensionalization of the basic equations, and the Reynolds number; Flow due to a moving body at small Reynolds number. **(15L)**

UNIT IV: Flow at large Reynolds number: The source of vorticity in motions generated from rest, Steady two-dimensional (2D) flow in converging or diverging channel; Boundary Layer Theory: Boundary layer equations, Laminar-turbulent flows, Boundary layer with pressure gradients, Oscillatory boundary layers; General features of irrotational flow due to a moving rigid body; 2D irrotational flow due to a moving cylinder with circulation; Impulsive motion of a fluid. **(15L)**

References:

1. F.M. White, Fluid Mechanics, 7th Edition, The McGraw-Hill Companies, Inc., 2011.
2. A.J. Chorin and J. E. Marsden, A Mathematical Introduction to Fluid Mechanics (Texts in Applied Mathematics), 3rd Edition, Springer-Verlag, 1993.
3. G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, 1st Edition, 1993.
4. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall of India Private Limited, S.I. Unit Edition, New Delhi, India, 1988.
5. L. D. Landau and E. M.Lifshitz, Fluid Mechanics (Volume 6 of Course of Theoretical Physics), Pergamon Press, 2nd Edition, 1987.

Advanced Theory of Differential Equations

Course Code: PHDAM2E Credits: 4

UNIT I: Review of existence and uniqueness solutions for linear system of first order differential equations: Homogeneous and nonhomogeneous linear systems; Linear differential equations with constant and periodic coefficients; Stability for linear systems of two equations; Local theory for nonlinear planar systems: Flow defined by a differential equation, linearization and stable manifold theorem, Stability and Lyapunov functions, Saddles, nodes, foci, centres-nonhyperbolic critical points. **(15L)**

UNIT II: Global theory for nonlinear planar systems: Limit sets and attractors, Poincare Benedixson theory; Saddle-node bifurcation by Lyapunov-schmidt reduction;1^{st}order PDEs inmore than two independent variables; Hamilton-Jacobi theory; Dirichlet's principle; General 2^{nd} order linear elliptic equations; Maximum-Minimum principle for the heat equation and for some parabolic equations. **(15L)**

UNIT III: First-order hyperbolic system: Linear and semilinear equations; Existence, Uniqueness and stability of the Cauchy problem for a linear hyperbolic system; Hyperbolic system of two first order quasilinear equations: Linearization of a reducible system of quasilinear equation by hodograph transformation; Diffusion equation and Brownian motion; Laplace's equation and Green's function. **(15L)**

UNIT IV: Separation of the variables; Orthogonality and characteristic functions; Series solutions of initial and boundary value problems; Nonlinear stability theory; Asymptotic approximation methods for Fourier integrals; Green's functions for bounded and unbounded regions; Variational properties and other methods; Propagation of discontinuities and singularities for hyperbolic equation; Illustrative examples from the fields of problems arisen from the fields of electromagnetic theory, heat flow, vibrations, gas dynamics, fluid dynamics, wave propagation, biology. **(15L)**

References:

1. P. Prasad and R. Ravindran, Partial Differential Equations, 2^{nd} Edition, New Age International (P) Ltd., Publishers, 2011.
2. C. Chicone, Ordinary Differential Equations with Applications, 2^{nd} ed. Springer,2006.
3. E.Zauderer, Partial Differential equations of Applied Mathematics, Pure and Applied Mathematics: A Wiley-Interscience Series of Texts, 3^{rd} Edition, John Wiley, 2006.
4. W. E. Boyce and R. C. DiPrima, Elementary Differential Equations, 8^{th} Edition, John Wiley and Sons, 2005.
5. L. C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, American Mathematical Society, 1998.
6. L. Perko, Differential Equations and Dynamical Systems (Texts in Applied Mathematics), 3^{rd} Edition, Springer-Verlag, New York Berlin Heidelberg, 1991.
7. E. A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, TMH Edition, Tata McGraw-Hill Publishing Company. Inc., New Delhi, 9^{th} Reprint, 1987.

Numerical Methods for Differential Equations

Course Code: PHDAM3E Credits: 4

UNIT I: Geometric numerical integration**:** Monotone equations and algebraic stability; Quadratic invariants to orthogonal flows; Hamiltonian systems; Nonlinear algebraic systems; Gaussian elimination for sparse linear systems; Iterative methods for sparse linear equations; Well-posed problems; Characteristic value problems; Stability of numerical methods; Error propagation and its controllability**.** **(15L)**

UNIT II: One-dimensional initial-value and boundary-value ordinary differential equations (ODEs)**:** General features of initial-value ODEs; The shooting collocation methods for nonlinear ODEs; Finite difference methods; Convergence and stability of the finite difference method; Nonlinear implicit finite difference equations; Higher order ODEs; General features of boundary-value ODEs; Higher order equilibrium methods; Eigen problems**.** **(15L)**

UNIT III: Elliptic, parabolic, and hyperbolic partial differential equations: Introduction to finite difference methods, Forward-Time Centered-Space (FTCS) methods, Backward-Time Centered-Space (BTCS) methods; Upwind methods; Control volume method; Spectral and conjugate gradient methods; Nonlinear equations and multidimensional problems, Asymptotic steady state solution to propagation problems**.** **(15L)**

UNIT IV: Finite element methods:Mathematical background; Construction of finite element space; Piecewise polynomial approximation; Weak solutions; elliptic, parabolic and hyperbolic problems; Time dependent problems; Transport problems; Finite element for unsteady problems; Iterative techniques for mixed methods; Finite element multigrid methods, Finite volume method**.** **(15L)**

References:

1. A. Iserles, A First Course in the Numerical Analysis of Differential Equations, 2nd Edition, Cambridge University Press, 2010.
2. J. D. Hoffman, Numerical Methods for Engineers and Scientists, Second Edition, CRC Press (Taylor and Francis Group), 2001.
3. S. C. Brenner and L. R. Scott, The Mathematical Theory of Finite Element Methods, Springer-Verlag, 1994.
4. C. Johnson, Numerical Solution of Partial Differential Equations by the Finite Element Method, Cambridge University Press, 1987.
5. G. D. Smith, Numerical Solution of Partial Differential Equations: Finite difference methods, Oxford University Press, 1977.

Sobolev Spaces and Partial Differential Equations

Course Code: PHDAM4E Credit: 4

Unit I: Distribution Theory: Introduction, operators on distributions, distribution with compact support, convolutions and Fourier transform of distribution, Schwartz space, tempered distributions. **(15)**

Unit II: Sobolev spaces: Weak derivatives, approximation by smooth functions, Extension theorem, Rellich's compactness theorem, Trace theorem, Poincare inequality, difference quotient. **(15)**

Unit III: Second order elliptic equations: Formulation of Dirichlet, Neumann problem, Weak formulation, Lax-Milgram lemma, Existence result, regularity up to the boundary, maximum principle, Eigenvalues and Eigen functions of symmetric elliptic operator. **(15)**

Unit IV: Linear Evolution Equations: Hyperbolic and Parabolic equations, existence of weak solutions, energy methods, maximum principles. **(15)**

References

1. L C Evans, Partial Differential Equations, Second Edition, AMS, 2010.
2. S Kesavan, Topics in Functional Analysis and Applications, Wiley Eastern, New Delhi, 1989.
3. D Gilbarg and N Trudinger, Elliptic Partial Differential Equations of second order, second edition, Springer, 1983.
4. G. Leoni, A First Course in Sobolev Spaces, AMS, 2009.
5. W Rudin, Functional Analysis, McGraw-Hill, New York, 1991.

Advanced topics in Fractional Differential Equations

Course Code: PHDAM5E Credit: 4

Unit I: Mittag-Leffler (ML) Function, Properties of ML function, Recurrence Relations and Operations on ML function, Laplace Transform of ML Function, Fractional Operators: Riemann-Liouville (RL) Fractional Derivative, Caputo Fractional Derivative and Riemann-Liouville Fractional Integral. Analysis on Fractional Operators, Relation in between RL and Caputo Fractional Derivative, Laplace Transform of Fractional Operators. (15)

Unit II: Existence and Uniqueness Theorem for Linear and Nonlinear Fractional Differential Equations (FDE). Successive approximation and Laplace Transform method to solve FDE in Homogeneous and Non homogeneous case. Stability Analysis. (15)

Unit III: Fractional Modelling: Physical models (Fractional Heat Equation, Fractional Wave Equation, Fractional Basset Equation, Fractional Bagley Torvik Equation), Biological models, System Biological models: Biochemical Kinetics, Enzyme Kinetics. (15)

Unit IV: Numerical Analysis: Numerical results of ML function, Approximation of Fractional operators, Numerical solution of linear and Nonlinear FDE: Adams Predictor Corrector method. Numerical solution for composite fractional differential Equations. (15)

References

1. K. Deithelm, The Analysis of Fractional Differential Equations, Springer, Berlin, 2010.
2. K.S.Miller and B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley, New York, 1993.
3. K. B. Oldham and J. Spanier, The Fractional Calculus, Academic Press, New York, 1974.
4. I. Podlubny, Fractional Differential Equations, Academic Press, New York, 1999.
5. S. G. Samko, A. A. Kilbas and O. I. Marichev, Fractional Integrals and Derivatives (Theory and Applications), Gordon and Breach Science Publishers, Amsterdam, 1993.
6. B. Ingalls, Mathematical Modelling in Systems Biology: An Introduction, MIT Press, Cambridge, 2013.