CENTRAL UNIVERSITY OF TAMIL NADU, THIRUVARUR

SCHOOL OF MATHEMATICS AND COMPUTER SCIENCES

M.Sc. STATISTICS AND APPLIED MATHEMATICS

COURSE STRUCTURE 2020-2021

Semester	Course code	Course title	Credits	Page No.
Semester 1	SAM011	Analysis I	4	2
	SAM012	Linear Algebra	4	3
	SAM013	Probability Distributions	4	4
	SAM014	Statistical Computing with "R" (Theory and Lab)	4	5
Semester 2	SAM021	Analysis II	4	6
	SAM022	Multivariate Statistical Analysis (Theory and Lab)	4	7
	SAM023	Numerical Analysis (Theory and lab)	4	8
	SAM024	Differential Equations	4	9
	SAM025	Statistical Inference	4	10
Semester 3	SAM031	Fluid Dynamics	4	11
	SAM032	Stochastic Processes	4	12
	SAM**E	Elective 1	4	13-32
	SAM**E	Elective 2	4	13-32
	SAM**E	Elective 3	4	13-32
Semester 4	SAM**E	Elective 4	4	13-32
	SAM**E	Elective 5	4	13-32
	SAM04P	Project work	8	
		Total credits	**72**	

List of Electives *(This list may be extended if needed)*

Sl. No.	Course code	Course title	Credits
1	SAM01E	Artificial Intelligence	4
2	SAM02E	Biostatistics	4
3	SAM03E	Calculus of Variations and Integral Equations	4
4	SAM04E	Design and analysis of experiments	4
5	SAM05E	Econometrics	4
6	SAM06E	Advanced Topics in Differential Equations	4
7	SAM07E	Mathematical Modelling in Biology	4
8	SAM08E	Integral Transforms	4
9	SAM09E	Machine Learning	4
10	SAM10E	Mechanics	4
11	SAM11E	Statistical Methods in Clinical Trials	4
12	SAM12E	Industrial Statistics	4
13	SAM13E	Advanced Numerical Methods	4
14	SAM14E	Time Series Analysis	4
15	SAM15E	Introduction to Cryptography	4
16	SAM16E	Computational Introduction to Number Theory	4
17	SAM17E	Regression Analysis	4
18	SAM18E	Generalized Linear Models	4
19	SAM19E	Introduction to Fractional Calculus	4

Syllabus in OBE Format
Department of Statistics and Applied Mathematics
School of Mathematics and Computer Sciences

A. Vision

Vision Statement of the Department

Vision Statement
To provide quality education to bring out potentialities in the students

B. Mission

Mission Statements of the Department

M1	To produce able statisticians and Applied Mathematicians
M2	To impart current knowledge on cross cutting issues in the domain
M3	To equip students with knowledge to work in domain specific targeted industries

C.Program Objective (PO)

After two years of successful completion of the program, the student will

PO1	Have a broad background in Statistics and Applied Mathematics, an appreciation of its various sub-disciplines and their inter-relationships, acquire in-depth knowledge about topics chosen from those offered through the department.
PO2	Be familiar with a variety of real-life situations where Statistics and Mathematics help accurately explain the underlying abstract or physical phenomena and able to recognize and appreciate the connections between theory and applications
PO3	Develop **the ability** to effectively and aptly use techniques from different sub-disciplines in solving a broad range of real-life problems.
PO4	Be Statistically and Mathematically **literate**. Graduates will recognize the importance and value of discipline.
PO5	Have the **versatility** to work effectively in a broad range of companies (including R&D sectors of financial, pharmaceutical, market research, software development companies, consultancy etc), or analytic, scientific, government, financial, health, teaching and other positions or continue for higher education.
PO6	Be able to independently read and analyze Statistical and Mathematical literature including survey articles, scholarly books, and online sources.
PO7	Be life-long learner, able to independently expand their expertise when needed, or out of their own interest.
PO8	Be able to exhibit ethical and professional behaviour in teamwork

D. Graduate Attributes for M.Sc. Statistics and Applied Mathematics Program

1. In-depth knowledge in sub-disciplines offered in this program
2. Problem Solving in various fields
3. Model Building and Data Analysis
4. Using Computational Software
5. Life-long learner
6. Ethical and professional behaviour in teamwork

E. PO to Mission Statement Mapping

	PO1	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PO8**
M1	3	3	3	3	3	3	3	3
M2	3	3	3	3	3	3	3	3
M3	3	3	3	3	3	3	3	3

E. Program Specific Outcomes (PSO)

On the successful completion of the program, the student will be able to

PSO1	Effectively recall basics and display knowledge of conventions.
PSO2	Develop stochastic and Mathematical models for studying real-life phenomena in diverse disciplines.
PSO3	Efficiently interpret and translate the outcomes obtained from the analysis of stochastic and mathematical models to an environment understandable to a layman.
PSO4	Effectively use necessary computational software.
PSO5	Apply statistical and mathematical techniques to optimize and monitor real-life phenomena related to industry and business analytics at local and global levels.

F. PSO to PO Mapping

	PSO1	**PSO2**	**PSO3**	**PSO4**	**PSO5**
PO1	3	3	3	3	3
PO2	3	2	2	2	3
PO3	3	3	2	3	3
PO4	3	3	2	3	3
PO5	3	3	3	3	3
PO6	3	2	2	1	2
PO7	3	2	2	3	3
PO8	1	3	3	1	3

SEMESTER - I					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM011	Analysis I	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Explain about the basic analysis on real line.	**Remember**
CO 2	Understand the convergence of sequences and series on real numbers and functions.	**Understand**
CO 3	Get knowledge on measures, measurable functions and the sequence of measurable functions and Lebesgue integration of simple and non-negative functions.	**Apply**
CO 4	Analyze the essence of proofs on Stone Weierstrass theorem, Fubini theorem, Fatou's lemma and convergence theorems.	**Analyze**
CO 5	Demonstrate the real-life applications by means of partitions on real line, sequence of functions and infinite series.	**Skill**

b. Syllabus

Units	**Content**	**Hrs.**
I	Sequences and Series of Real Numbers**:** Convergent and Divergent of Sequence, Cauchy Sequence, Upper Limit and Lower Limit of Real Sequences. Cauchy Criterion for Series of Numbers, Absolute Convergence, Series of Non-Negative Real Numbers, Geometric Series, The number e, Cauchy Product of Series, Merten's Theorem, Rearrangement of Series, Riemann's Theorem on Rearrangement of Series, Riemann-Stieltjes Integral: Definition, Existence of the Integral, Properties of the Integral, Integration and Differentiation, Rectifiable Curve**.**	20
II	Sequences of Functions**:** Point-wise Convergence, Uniform convergence, Uniform Convergence and Continuity, Uniform Convergence and Integration, Uniform Convergence and Differentiation, Equi-continuous Family of Functions, Stone-Weierstrass Theorem**.**	10
III	Measure**:** Length of Open and Closed Set Inner and Outer Measure, Measurable Sets, Regularity, Borel and Lebesgue Measurability, Abstract Measure, Extension of a Measure, Completion of a Measure. Measurable Functions: Simple Measurable Functions, Sequence of Measurable Functions and their convergence**.**	15
IV	Lebesgue Integration**:** Integrals of simple functions, Integrals of Non-Negative Functions, Fatou's Lemma, Monotone Convergence Theorem, Dominated Convergence Theorem, Integration of Series, Riemann and Lebesgue Integrals, Product Measure, Fubini's Theorem**.**	15
	References: **1.** W**.** Rudin, Principles of Mathematical Analysis, 3rd Edition, McGraw-Hill, 1984**.** **2.** T**.** Apostol, Mathematical Analysis, 2nd Edition, Narosa Publishing House, 1985**.** **3.** G**.** de Barra, Measure Theory and Integration, New Age	

	International (P) Limited, 1996. **4.** H.L.Royden, Real Analysis, 3rd Edition, McMillan Publication Co. Inc., 1988. **5.** Billingsley P. Probability and measure. John Wiley & Sons; 2008.	

c. Mapping of Program Specific Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	0	2	0	1
CO2	3	2	2	0	1
CO3	3	2	2	1	1
CO4	3	2	1	1	1
CO5	3	2	2	2	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 10 x 1 = 10 marks)**	2	2	2	2	2
Part – B **(Short Answer - 5 x 4 = 20 marks)**	10	10	-	-	-
Part – C **(Essay- 3 x 10 = 30 marks)**	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

i.Model Question Paper

S.No.	Model Questions	Knowledge	Level
	Part A: Objective Type **Multiple choice 10 x 1 = 10**		
1	Evaluate the supremum and infimum for the sequence (a_n) be a sequence with $a_n := \begin{cases} 2, & \text{if n is even} \\ \text{lowest prime} \\ \text{factor of } n, & \text{if n is odd} \end{cases}$ (a) 1, ∞ (b) 2, ∞ (c) 0, ∞ (d) 1, 2	Evaluate	Remember
2	Evaluate the liminf and limsup for the sequence (a_n) with $a_n = \sin\frac{n\pi}{2} + \frac{(-1)^n}{n}$. (a) 1, -1 (b) -1, 1 (c) 0, 1 (d) -1, 0	Evaluate	Remember
3	Choose the correct option. There is a real valued function defined on $[a, b]$ which is differentiable but not an integrable function. A. True B. False C. Partially true. D. None of the above	Describe	Remember
4	Derive the sum of the series $\sum_{n=1}^{\infty} \frac{\pi^2}{n(n+1)} - \frac{1}{2^n}$. (a) $\pi - 1$ (b) $\pi^2 - 1/2$ (c) $\pi - 1/2$ (d) $\pi^2 - 1$.	Evaluate	Remember
5	Determine the value of $a \in \mathbb{R}$ such that the sequence of function x^n, $x \in [0, a]$ converges uniformly. (a) 1 (b) $a \in (0, 1)$ (c) $a \in (0, 1]$ (d) No such value exist	Determine	Remember
6	Determine the values of $x \in \mathbb{R}$ such that the series $\sum_{k=1}^{\infty} x^k k^k$ converges. (a) 0 (b) 1/2 (c) 1 (d) No such value exist.	Determine	Remember
7	Pick the wrong topology of $X := \{a, b\}$ (a) $\tau_1 = \{\Phi, \{a, b\}\}$ (b)$\tau_2 = \{\Phi, \{a\}, \{a, b\}\}$ (c) $\tau_3 = \{\Phi, \{a\}, \{b\}, \{a, b\}\}$ (d) None of these.	Example	Remember
8	Pick the zero measure set. (a) ϵ- neighbourhood (b) interval in $\mathbb{R}$ (c) Uncountable set in $\mathbb{R}$ (d) (b) and (c).	Example	Remember
9	For vitali set $V \subset [0, 1]$, evaluate the integral $\int_{\mathbb{R}} \chi_V dm$ is (a) ∞ (b) 1 (c) Undefined (d) 0	Evaluate	Remember
10	Pick the one which does not belong to the space of Lebesgue integrable function (a) $\int_{\mathbb{R}} \cos x \; dm$ (b) $\int_{[0,\pi/2]} \cos x \; dm$ (c) $\int_{[0,\pi]} \sin x \; dm$ (d) (b) and (c).	Evaluate	Remember

S.No.	Model Questions	Knowledge	Level
	PART B Short Answer **The answer should not exceed 200 words 5 x 4 = 20**		
11	a) Explain the concept of uniformly continuous function. (or) b) Explain the Riemann Stieltjes integral	Explain	Remember
12	a)Discuss the relation between the succesive approximations of first order linear IVP to the sequence of functions and the convergence as well. (or) b) Describe the significance of Lebesgue integrable functions in real life applications	Discuss Describe	Skill
13	a) Discuss the Almost Everywhere (a.e) Convergence. (or) b) Explain the ϵ - neighbourhood and its measure.	Discuss Explain	Understand
14	a) Describe the lebesgue outer measure of an interval. (or) b) Explain the Dirichlet function and its measurability.	Describe Explain	Apply
	PART C Essay Answer **The answer should not exceed 400 words 3 x 10 = 30**		
15	a) i) Discuss the convergence and find the limit for the sequence $3, 3+\frac{1}{3}, 3+\frac{1}{3+\frac{1}{3}}, 3+\frac{1}{3+\frac{1}{3+\frac{1}{3}}}, \cdots$ ii) For $n \in \mathbb{N}$, let (a_n) with $a_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2}$. Show that the sequence (a_n) is convergent. (or) b) i) Discuss the absolute convergence for the following series (1). $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ (2) $\sum_{n=1}^{\infty} (-1)^n \frac{e^{-nx}}{n}$. ii) Show that the series $\frac{1}{(1+a)^p} - \frac{1}{(2+a)^p} + \frac{1}{(3+a)^p} - \cdots$, $a > 0$. converges absolutely for $p > 1$ and conditionally for $0 < p \leq 1$.	Discuss	Understand

S.No.	Model Questions	Knowledge	Level
16	a) i) Discuss the convergence of the following sequence (1.)](a_n) with $a_n := (n!)^{1/n}$. (2.)$(b_n^{1/n})$ with $b_n = \frac{(3n)!}{(n!)^3}$ ii) Let $\sum_{n=1}^{\infty} f_n(x),\ f_n(x) := \frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2},\ x \in [0,1]$. Show that at $x = 0$, $\frac{d}{dx}\sum f_n \neq \sum \frac{df_n}{dx}$ (or) b). i) Let (a_n) be a sequence of non-negative terms such that 1) $1 \leq a_n \leq n$ for all $n \in \mathbb{N}$ 2) $a_n \leq M$ for all $n \in \mathbb{N}$. Both the cases show that $(1 + a_n)^{1/n} \to 1$. (ii)]Give an example to show that the sequence of rational numbers converges to an irrational number.	Discuss Example	Understand
17	a) i) Let f be a non negative real valued measurable function on $(X, \mathcal{A}, \mu)$. Then the following result hold. $\int_X f d\mu = 0$ if and only if $f = 0$ a.e. ii) If f is real valued measurable function defined on $(X, \mathcal{A})$ then show the following: (1) $f + g$, fg and $\sup(f, g)$ are measurable. (or) b) i) Suppose $f : [a, b] \to [0, \infty)$ is a Riemann integrable function. Then f is measurable with respect to $\mathcal{M}$ and $\int_a^b f(x)dx = \int_{[a,b]} f dm$ ii) State and prove the Monotone Convergence Theorem.	Discuss	Analyse

SEMESTER - I					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM012	Linear Algebra	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	Level
s	Explain the basic concepts of vector spaces, including bases and dimensions, matrix representations of linear transformations and their properties, and inner products of vector spaces.	**Remember**
CO 2	Createan orthonormal set from a set of linearly independent vectors by employing the Gram-Schmidt orthogonalization process.	**Understand**
CO 3	Analyze the existence and uniqueness of the solution of a linear system in matrix form as $AX = B$ by the Gauss elimination method.	**Apply**
CO 4	Examine the use of bilinear and quadratic forms in the geometric study of conic and quadratic curves.	**Analyze**
CO 5	Understand the decomposition of a vector space into the Jordan form and Sylvester's law for congruent matrices and also their applications.	**Skill**

b. Syllabus

Units	Content	Hrs.
I	Vector Spaces; Subspaces; Linear combinations and span, Linear dependence and independence, Bases and dimension of a vector space; Vector spaces with inner products; Gram-Schmidt orthogonalization; Linear Transformations (**LT**) and its properties; Representation of transformations by matrices; Invertibility and isomorphism; Change of bases; Orthogonal transformation; Dual spaces**.**	15
II	Systems of linear equations **-** theory and computational aspects; Homogeneous and non-homogeneous systems of linear equations; Existence and uniqueness of solutions; Matrices and elementary row operations; Row-reduced echelon matrices; Gaussian elimination method; Rank of a matrix; Inverses, G-inverse and transposes of a matrix**.**	15
III	Eigen values and Eigen vectors of an **LT**; Diagonalization of **LT**; Properties of Eigen values and Eigen vectors; Cayley-Hamilton theorem and its applications; Minimal polynomial for an **LT**; Eigen values of matrix polynomials; Matrix limits and Markov chains; Operators on complex and real vector spaces; Orthogonal projections and the spectral theorem; Similar linear transformations; Positive definite matrices and least squares.	15
IV	Bilinear forms; Quadratic forms; Rank, index, and signature of quadratic forms; Reduction of quadratic form into a canonical form; Decomposition of a vector space into the Jordan form; Single value decomposition and its applications; Sylvester's law for congruent matrices; Its application in Einstein's special theory of relativity;	15

	Generalized Eigen value problem; Method of Lagrange multipliers.	
	References: 1. G. Strang, Linear Algebra and its Applications, 4th Edition, Cengage Learning India Pvt Ltd., 2005. 2. Ramachandra Rao, A. and Bhimasankaram, P. (2000). Linear Algebra. Hindustan Book Agency 3. S. H. Friedberg, A. J. Insel, and L. E. Spence, Linear Algebra, 4th Edition, Prentice-Hall of India, 2003. 4. Searle, S. R. (1982). Matrix Algebra Useful for Statistics, John Wiley, New York 5. S. Axler, Linear Algebra Done Right (Undergraduate texts in Mathematics), Springer, 2nd Edition, 1997. 6. Lay, D. C. Lay, S. R. and Mc Donald, J. J. (2016). Linear Algebra and Its Applications, Fifth Edition, Pearson, Boston.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	2	1	1	1
CO2	3	1	1	1	1
CO3	3	2	2	2	2
CO4	3	2	2	0	0
CO5	3	2	1	0	1

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 10 x 1 = 10 marks)**	2	2	2	2	2
Part – B **(Short Answer - 5 x 4 = 20 marks)**	10	10	-	-	-
Part – C **(Essay- 3 x 10 = 30 marks)**	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

Sl. No.	Model Questions	Specification	Level
	Part – A: Objective Type **Multiple Choice Marks: 10 x 1 = 10**		
1	Let V be a finite dimensional vector space. If V^* is the dual of V, then --- (A)$\dim V > \dim V^*$,(B)$\dim V < \dim V^*$,(C)$\dim V = \dim V^*$,(D)None of these	Recall, Check	Remember, Understand
2	Let T be a linear operator on a finite dimensional vector space V and $\lambda_1, \lambda_2, . . . , \lambda_k$ be the distinct characteristic values of T. Let W_i be null spaces of $(T - \lambda_i I)$. If T is diagonalizable, then ---------- (A) $(\dim W_1)(\dim W_2)(\dim W_3) . . . (\dim W_k) = \dim V$ (B) $\dim W_1 + \dim W_2 + \dim W_3 + . . . + \dim W_k = \dim V$ (C) $\dim W_1 - \dim W_2 - \dim W_3 - . . . - \dim W_k = \dim V$ (D) None of these	Recall, Check, Examine	Remember, Understand, Analyze
3	Which of the following condition is true for the quadratic form $Q = x^T A x$ is positive definite, where $A = [a_{ij}]$ and $1 \le i, j \le n$. (A) $a_{ii} < 0$ (B) $a_{ij} > 0$ (C) $a_{ii} > 0$ (D) None of these	Check, Examine	Understand, Analyze
4	Which of the following matrices are the right inverses of matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$? (A) $\begin{pmatrix} 1 & -2 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 2 & -5 \\ -1 & 3 \end{pmatrix}$ (B) $\begin{pmatrix} -1 & 2 \\ 0 & -1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ -2 & 5 \\ 1 & -3 \end{pmatrix}$ (C) $\begin{pmatrix} -1 & -2 \\ 0 & -1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ -2 & -5 \\ -1 & -3 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 2 & 5 \\ 1 & 3 \end{pmatrix}$	Verify, Examine	Analyze, Understand
5	The following vectors $\left(\frac{1}{4}, 0, -\frac{1}{4}\right), \left(\frac{1}{3}, -\frac{1}{3}, 0\right)$ and $\left(0, \frac{1}{2}, -\frac{1}{2}\right)$ are --------- (A) Linearly dependent, (B) Linearly independent, (C) Constant, (D) None of these	Check, Recall	Analyze, Understand
6	If m, n are the rank and number of linearly independent columns vector of the matrix A, then which of the following is correct (A) $m > n$ (B) $m < n$ (C) $m \le n$ (D) $m \ge n$	Check, Recall	Understand, Remember
7	If rank of matrix A is 5 and nullity of A is 3, then the order of the matrix is ----- (A) 3 (B) 5 (C) 8 (D) Zero	Recall, State	Apply, Skill
8	Which of the following condition is correct for solution of the system, $Ax = b$ (A) $Rank\ A \neq rank[A; b] = n$ (B) $Rank\ A = rank[A; b] \neq n$ (C) $Rank\ A = rank[A; b] = n$ (D) $Rank\ A \neq rank[A; b] \neq n$	Check, Recall	Remember, Analyze
9	The characteristic equation for the unitary matrix of the matrix, $A = \begin{bmatrix} 3+4i & -5i \\ -7 & 6-2i \end{bmatrix}$, is --------- (A)$\lambda^2 - i\lambda - 1 = 0$, (B) $\lambda^2 + i\lambda = 0$, (C) $\lambda^2 + i\lambda + 1 = 0$, (D) None of these	Find	Apply, Skill

10	If A is symmetric matrix of rank 'r' then A can reduce to ----- (A) Diagonal matrix with exactly r, non-zero elements. (B) Diagonal matrix with exactly $< r$, non-zero elements. (C) Identity matrix (D) None of these	Recall, Check	Remember, Understand
	PART – B Short Answer **The answer should not exceed 200 words Marks:5 x 4 = 20**		
11	**a)** Prove that a non-empty subset W of vector space $V(F)$ is a subspace of $V(F)$ if and only if for each pair of vectors x, y in W and each scalar α in a field F, the vector $(\alpha x + y)$ is again in W. **(or)** **b)** Check whether the following system of linear equations is consistent or inconsistent by employing Gaussian elimination method. Also find the complete solution set, and if the solution set is infinite, specify three particular cases. $-5x_1 - 2x_2 + 2x_3 = 14,$ $3x_1 + x_2 - x_3 = -8,$ $2x_1 + 2x_2 - x_3 = -3.$	Prove, Check, Verify, Examine	Remember, Understand, Apply, Analyze, Skill
12	**a)** Define rank-factorization of a matrix. Also, find a rank-factorization of the following the matrix $A = \begin{pmatrix} 2 & 4 & 1 & -1 \\ 3 & 6 & 0 & 1 \\ -1 & -2 & -2 & 3 \end{pmatrix}$. **(or)** **b)** Let A B, C be three banks in a certain town are competing for investors. Currently, Bank A has 40% of the investor, Bank B has 10%, and Bank C has the remaining 50%. Assuming the townsfolk are tempted by various promotional campaigns to switch banks. Records show that each year Bank A, B, and C keep half, two-third, and half of their investors, with the reminder switching equally to Banks B and C, Bank A and C, and Bank A and B respectively. Find the distribution of investors after two years.	Define, Examine, Recall	Understand, Apply, Analyze, Skill
13	**a)** Prove that every matrix A of order $n \times n$ is similar to an upper triangular matrix over the field of complex numbers. **(or)** **b)** Let V be a finite-dimensional inner product space and let T be a linear operator on V. Then prove that there exists a unique function $T^*: V \to V$ such that $< T(x), y > = < x, T^*(y) >$ for all $x, y \in V$. Furthermore, T^* is linear.	Prove, Define, Recall	Understand, Remember, Analyze
14	**a)** If $A = \begin{pmatrix} 3 & -6 & 0 \\ -6 & 0 & 6 \\ 0 & 6 & -3 \end{pmatrix}$, then show that A is orthogonally diagonalizable by finding an orthogonal matrix $'U'$ and a diagonal matrix $'D'$ such that $A = UDU^T$. **(or)** **b)** Write short notes on definiteness of two variable quadratic forms.	Prove, Find, Define, State	Understand, Remember, Examine, Apply, Skill
	PART – C Essay Answer **The answer should not exceed 400 words Marks: 3 x 10 = 30**		

15	**a)** If $A \subseteq S$, where S is a subspace of a vector space V, then prove the following statements are equivalent**:** **Statement 1:**A is minimal generating set of S**.** **Statement 2:** Every element of S can be expressed uniquely as a linear combination from the set A**.** **Statement 3:** A generates S and A is linearly independent. **(or)** **b)** Construct an orthogonal basis set (T) for the subspace $W = span(B)$ of vector space $\mathbb{R}^4$, where $B = \{[2,1,0,-1],[1,0,2,-1],[0,-2,1,0]\}$ by applying Gram-Schmidt process**.** Also, find an orthogonal basis C for $\mathbb{R}^4$ that contains the orthogonal set T by Enlarging method.	Recall, Check, Define, Find	Understand, Remember, Apply, Analyze, Skill
16	**a)** For any two matrices A and G, prove the following statements are equivalent**:** **Statement 1:**G is an $g-$ inverse of A**, Statement 2:**$AGA = A$**,** **Statement 3:**AG is idempotent and $rank(AG) = rank(A)$**.** **Statement 4:**GA is idempotent and $rank(GA) = rank(A)$**.** **(or)** **b)** Define normal and self-adjoint operators for linear transformations. If T is a self-adjoint operator on a finite**-**dimensional inner product space V, then prove that **i)** every eigenvalue of T is real, and **ii)** the characteristic polynomial of T splits when V is a real inner product space**.**	Define, Prove, Examine.	Remember, Understand, Analyze
17	**a)** State and prove the singular value decomposition theorem for linear transformations**.** **(or)** **b)** Define bilinear form over a vector space $V(F)$**.** If the bilinear form $H: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ is defined by $H\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right) = 2a_1b_1 + 3a_1b_2 + +4a_2b_1 - a_2b_2$ for $[a_1, a_2]^T$, $[b_1, b_2]^T \in \mathbb{R}^2$ and β is the standard ordered basis for $\mathbb{R}^2$, then verify H is a bilinear form. Also, find the matrix representation of H with respect to β**.**	Sate, Prove, Define, Find	Understand, Remember, Analyze, Apply, Skill

SEMESTER - I					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM013	Probability Distributions	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	Level
CO 1	Recall the pdf, pmf, cdf, moments, and generating functions of a random variable.	**Remember**
CO 2	Explain standard discrete and continuous distributions.	**Understand**
CO 3	Explain and apply moment inequalities to obtain bounds on entity of interest.	**Apply**
CO 4	Derive distribution of function of random variables.	**Analyze**
CO 5	Separate mixture of distributions	**Skill**

c. Mapping of Program Specific Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	2	1	1	1
CO2	2	3	2	1	2
CO3	2	3	2	3	3
CO4	2	3	2	1	3
CO5	1	2	3	1	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Short Answer- 6 x 2 = 12 marks)**	4	4	2	2	2
Part – B **(Long Answer -4 x 12 = 48 marks)**	8	8	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

g. Rubric for Assignments

Sl. No.	Criteria	100%	75%	50%	25%	0%	Relation to COs
1	**Content** **50%**	Ideas are detailed, well developed, supported with specific evidence &facts andexamples	Ideas aredetailed, Developed and supported with evidence and facts mostly specific.	Ideas are presented but not particularly developed or supported;	Content is not sound	Not attended	CO1, CO2, CO5
2	**Organiza -tion** **50%**	Includes title, introduction, statement ofthe main idea withillustration and conclusion.	Includes title, introduction, statement ofmain ideaand conclusion.	organizational tools are weak or missing	Noorganization	Not attended	CO1, CO2, CO5

h. Rubric for Seminar

Sl. No.	Criteria	100%	75%	50%	25%	0%	Relation to COs
1	**Knowledge and Understanding** **50%**	Exceptional knowledge of facts, terms, and concepts	Detailed knowledge of facts, terms, and concepts	Considerable knowledge of facts, terms, and concepts	Minimal knowledge of facts, terms, and concepts	Not Attended	CO3, CO4
2	**Presentation** **50%**	Well Communicated with logical sequences, examples, and references	Communicated with sequences	Just Communicated	No coherent communication	Not Attended	CO3, CO4

SAM013 - Probability Distributions

Sl. No.	Model Questions	Specification	Level
	Part – A: Short Answer (Attempt any SIX) **Marks: 6 x 2 = 12**		
1	Give axiomatic definition of probability.	Recall	Remember
2	State Bayes' Theorem for probability.	Recall	Remember,
3	What is pairwise independence?	Recall,	Remember,
4	Define cumulative distribution function. State its properties.	Recall, State	Remember, Understand
5	Define conditional probability.	Recall	Remember
6	Check whether following function defines a probability density function or not: $$f(x) = \frac{1}{2}e^{-\lvert x\rvert}, -\infty < x < \infty.$$	Check	Analyze, Apply
7	Define convergence in distribution.	Define	Remember
8	Let $X \sim N(0,1)$. Find distribution of $Y = X + 2$.	Find	Apply, skill
	PART – B Long Answer (Attempt any FOUR) Marks: 4 x 12=48		
9	a) Define Expectation and Variance of a random variable. Find the same when random variable X follows $B(n,p)$ distribution. b)Given that $F(x)$ and $G(x)$ are distribution functions, verify whether the following are distribution functions or not: (i) $F(x) + G(x)$, (ii) $F(x).G(x)$	Define, Find, Verify	Remember, Understand, Analyze, Apply, skill
10	a) Explain the following terms: (i) Joint distribution, (ii) Marginal distribution, (iii) Conditional distribution. b) For the following bivariate PMF, $$P\{X = x, Y = y\} = \frac{(x+y+k-1)!}{x!\,y!\,(k-1)!} p_1^x p_2^y (1 - p_1 - p_2)^k,$$ where $x, y = 0,1,2,\ldots; k \geq 1$ is an integer; $0 < p_1 < 1; 0 < p_2 < 1$ and $p_1 + p_2 < 1$, find the marginal PMF of X and the conditional distribution of $Y\|X = x$.	Explain, Find	Remember, Understand, Analyze, Apply, skill
11	a) For the random variable with PDF $f(x;\lambda) = \frac{e^{-x}x^{\lambda}}{\lambda!}, x > 0, \lambda \geq 0$ is an integer, show that $P\{0 < X < 2(\lambda+1)\} > \frac{\lambda}{\lambda+1}$. b)Define moment generating function (MGF). Derive MGF of X, where $X \sim Poisson(\lambda)$. Hence, obtain $E(X)$ and $V(X)$.	Prove, Define, Obtain	Remember, Understand, Analyze, Apply, skill
12	a) Decompose the following distribution function:	Problem, Prove	Understand, Analyze, Apply, skill

	$$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < \frac{1}{2} \\ 1, & x \ge \frac{1}{2}. \end{cases}$$ b) Let X_1, X_2 be iid r.v's with common PMF $P\{X = \pm 1\} = 1/2$. Write $X_3 = X_1 X_2$. Show that X_1, X_2, X_3 are pairwise independent but not mutually independent.		
13	a) Let $X \sim Exponential(\lambda)$. Find the distribution of $Y = [X]$, where $[X]$ denotes largest integer not larger than X. b)Let X_1, X_2, X_3 be i.i.d random variables with common PDF $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & o.w. \end{cases}$ Find the joint distribution of $Y_1 = X_1 + X_2 + X_3$, $Y_2 = \frac{X_1}{X_1+X_2}$ and $Y_3 = \frac{X_1+X_2}{X_1+X_2+X_3}$.	Find, Problem	Analyze, Apply, skill
14	Write short notes on any **THREE** of the following: (3 x 4 =12) a) P{X<Y} in a continuous bivariate distribution. b) Convergence in Probability c) Chebychev's Inequality d) Continuous Uniform distribution e) Weak law of large numbers and Central limit theorem.	Note	Remember, Understand, Analyze, Apply, skill

SEMESTER - I					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM014	Statistical Computingwith "R" (Theory and Lab)	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	Level
CO 1	Learning basic statistical tools, types of qualitative and quantitative data	Remember
CO 2	Diagrammatic and graphical representations and organize, manage and present data.	Understand
CO 3	Data manipulation, plot the graphs and charts with the help of computing features in R Programming. The given data Interpretation with different distribution functions. Demonstrate Statistical functions in R	Apply
CO 4	The relevance and importance of the theory in solving practical problems in the real-world using R programming.	Analyze
CO 5	Statistical inference,contingency tables, correlation, regression, generalized linear models, advanced modeling methods	Skill

b. Syllabus

Units	Content	Hrs.
I	Introduction to R - A programming language and environment for data analysis and graphics. Syntax of R expressions: Vectors and assignment, vector arithmetic, generating regular sequence, logical vector, character vectors, Index vectors; selecting and modifying subsets of data set, Data objects: Basic data objects, matrices, partition of matrices, arrays, lists, factors and ordered factors, creating and using these objects; Functions- Elementary functions and summary functions, applying functions to subsets of data.	15
II	Data frames: The benefits of data frames, creating data frames, combining data frames, Adding new classes of variables to data frames; Data frame attributes. Importing data files: import. Data function, read. Table function; exporting data: export. data function, cat, write, and write. Table functions; outputting results - sink function, formatting output - options, and format functions; Exporting graphs - export. graph function.	15
III	Random numbers from various distributions like uniform, Normal, gamma, exponential, beta, F, Poisson, binomial, etc Graphics in R: creating graphs using plot function, box plot, histogram, line plot, stem and leaf plot, pie chart, bar chart multiple plot layout, plot titles, formatting plot axes. Interactively adding information of plot - Identifying the plotted points, adding trend lines to current scatter plot, adding new data to current plot, adding text and legend.	15
IV	Loops and conditional statements: Control Statements; if statement, if else Statement. Looping statement; for loop, repeat, while loop Developing simple programs in R for data analysis tasks, saving programs, executing stored programs, defining a new binary operator, assignment within function, more advanced examples, object-oriented	15

	programme. Creating function libraries- library function, attaching and detaching the libraries.	
	References: **1.** J.M. Chambers, Programming with Data: A guide to S language, Springer,1998. **2.** W.N. Venables and B. D. Ripley, S Programming, Springer, 2000. **3.** B. S. Everitt, A handbook of Statistical Analysis using S-Plus, Chapman & Hall, 1994. **4.** P. Dalgaard, Statistics and computing: Introductory Statistics with R, Springer, 2002. **5.** J. Maindonald and J. Braum, Data Analysis and Graphics Using R: An example-based approach Second Edition, Cambridge Series in Statistical and Probabilistic Mathematics, 2007.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	3	3	3	2
CO2	3	3	3	3	3
CO3	3	3	3	3	3
CO4	2	2	1	3	2
CO5	3	1	1	1	2

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Short Answer- 6 x 2 = 12 marks)**	4	4	2	2	2
Part – B **(Long Answer -4 x 12 = 48 marks)**	8	8	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

	Model Questions	Specification	Level
	Part – A: Short Answer (Attempt any SIX) **Marks: 6 x 2 = 12**		
1	What is the output of x <- "Students" typeof(x)	Recall, Find	Analyze, Apply
2	List the basic data structures in R.	Recall	Remember,
3	Write the syntax for printing a matrix of order 2 * 2	Recall, State	Analyze, Apply,
4	List the measures of central tendency and dispersion	Recall, State	Remember, Understand
5	Write down the different types of interactive visualization for a given data.	Recall, State	Remember
6	List down the different control statements in R programming	Recall, State	Analyze, Apply
7	What are the different types of probability distributions	Define	Remember
8	List down the different control statements in R programming	Recall, State	Apply, skill
	PART – B Long Answer (Attempt any FOUR) Marks: 4 x 12=48		
9	a) Explain the different data types and data frame functions in R. b) Write in detail about the object attributes and elementary functions in R.	Define, Explain	Remember, Understand, Analyze, Apply, skill
10	a) Explain the different matrix computations used in R with examples along with the syntax b)Explain the concept of importing and exporting data in R.	Explain,Obtain	Remember, Understand, Analyze, Apply, skill
11	a) Explain the basic probability distributions in R b) Write R code to print Fibonacci series	Define, Obtain	Remember, Understand, Analyze, Apply, skill
12	a)Write the general syntax for loop and control statements in R b) Explain the in-built function to generate Normal distribution	Explain,obtain	Understand, Analyze, Apply, skill
13	a)Write R code for finding the greatest of three numbers using control statements b) Explain the different types of library functions in R	Explain, Problem	Analyze, Apply, skill
14	Write short notes on any **THREE** of the following: (3 x 4 =12) a) Types of arithmetic operators and logical operators in R b) Lists, factors and arrays c) Create a table of all details with M.Sc class students marks in first semester using R d) Random numbers generation using Poisson distribution in R e) To test the hypothesis using ANOVA	Note	Remember, Understand, Analyze, Apply, skill

SEMESTER - II					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM021	Analysis II	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Explain different normed linear spaces, operators and discuss the concepts of complex numbers	Remember
CO 2	Understand the basic context of finite and infinite dimensional spaces and the analytic function.	Understand
CO 3	Work on operators, functionals, contour Integration and Cauchy residue theorem	Apply
CO 4	Analyze the fundamental theorems on Banach spaces, Hilbert spaces and determine the nature of the singularities	Analyze
CO 5	Solve mathematical problems by the knowledge of functional analysis and complex analysis.	Skill

b. Syllabus

Units	**Content**	**Hrs.**
I	Complex Numbers, geometric representation, powers and roots of complex numbers. Functions of a complex variable. Analytic functions. Cauchy-Riemann equations. Elementary functions. Conformal mapping (for linear transformation), Contours and contour integration. Cauchy's theorem, Cauchy integral formula. Power Series, term by term differentiation, Taylor series, Laurent series, Zeros, singularities, poles, essential singularities, Residue theorem and its applications.	20
II	Banach spaces, Riesz Lemma (On compactness of the unit ball in a normed linear space), Bounded linear maps on finite and infinite-dimensional normed linear spaces: Hahn Banach Theorem (geometric and extension forms), characterization of finite-dimensional normed linear spaces, Fundamental theorems on Banach Spaces-Uniform Boundedness Principle, Closed Graph Theorem, Open Mapping Theorem.	15
III	Dual spaces of some classical spaces, Hilbert spaces: Gram-Schmidt orthonormalization process, Bessel's inequality, orthonormal basis, Riesz Representation Theorem-Dual of a Hilbert space.	10
IV	Bounded operators on a Hilbert space: Adjoin of an operator, orthogonal projections, self-adjoin, normal and unitary operators, Introduction to Banach Algebras-Spectrum of an operator, Spectral Theorem for finite dimensional Hilbert spaces.	15
	References: **1.** W. Brown and R. V. Churchill, Complex Variables and Applications, McGraw-Hill, 2004. 2. L. V. Ahlfors, Complex Analysis, McGraw-Hill, 1966. **3.** B.V. Limaye, Functional Analysis, 2^{nd} Edition, New Age International, 1996. **4.** E. Kreyzig, Introduction to Functional Analysis with	

	Applications, **Wiley, 2007.** **5.** G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963. **6.** A.E. Taylor and D.C. Lay, Introduction to Functional Analysis, Second Edition, Wiley, 1980.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	1	2	1	2
CO2	3	2	2	1	2
CO3	3	2	2	1	2
CO4	3	2	1	1	2
CO5	3	1	2	1	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 10 x 1 = 10 marks)**	2	2	2	2	2
Part – B **(Short Answer - 5 x 4 = 20 marks)**	10	10	-	-	-
Part – C **(Essay- 3 x 10 = 30 marks)**	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

Sl. No.	Model Questions	Specification	Level
	Part – A: Objective Type **Multiple choice 10 x 1 = 10**		
1	Choose the incorrect statement. A. Norms are equivalent in infinite dimensional spaces. B. Closed subset of a Banach space is Banach. C. Dense subset of a Banach space is not Banach. D. $l^2((\mathbb{N}))$ is the only inner product space on $l^p((\mathbb{N}), 1 \leq p \leq \infty$.	Recall	Remember
2	Choose the correct inclusions. **A.** $c_0 \subset c \subset l^\infty$ C. $c_0 \subset l^\infty \subset c$ B. $l^\infty \subset c \subset c_0$ D. None of the above	Recognize	Remember
3	State the condition(s) in which a continuous map becomes open. A. In Banach spaces B. Surjective C. A and B D. Infinite dimensional spaces	Recognize	Remember
4	Choose the incorrect statements. A. Null space of a closed linear operator is not closed. B. Inverse of a closed operator is closed. C. Let L and R be left and right shift operators on $l^2(\mathbb{N})$, then $L^* = R$ D. A is normal if and only if $AA^* = A^*A$.	Recognize	Remember
5	Choose the incorrect(s) statement. A. Unit ball in infinite dimensional normed linear space is compact. B. The space $l^\infty((\mathbb{N})$ is not separable. C. The space $\mathbb{C}^n$ is reflexive. D. Every Hilbert space is reflexive.	Recognize	Remember
6	Evaluate the residue of f at the isolated singular point in the upper half plane is A. $\frac{-1}{2e}$ B. $\frac{1}{2e}$ C. $\frac{-1}{e}$ D. $\frac{1}{e}$	Evaluate	Remember
7	Let C be the anti-clockwise oriented circle centred at i of radius 2. Evaluate the contour integral $\oint \frac{dz}{z^4-1}$. $A. -\frac{\pi}{2}$ B. $\frac{\pi}{2}$ C. $-\pi$ D. π	Evaluate	Skill
8	Given a real number a > 0, consider the triangle with vertices 0; a; a + ia. If the triangle is given in the anticlockwise orientation; Then evaluate the contour integral $\oint_\Delta Re(z)dz$ A. $\frac{ia^2}{2}$ B. ia^2 C. 0 D. $a^2/2$	Evaluate	Skill
9	Identify the value of $(1 + i\sqrt{3})^{33}$ A. $(-2)^{33}$ B. $(2)^{33}$ C. $(-4)^{33}$ D. $(4)^{33}$	Identify	Remember
10	Discuss the convergence for the sequence $\left(\frac{1}{n} + i^n\right),\ n \in \mathbb{N}$ A. Divergent B. Convergent C. It has multiple limits D. A and C	Identify	Remember
	PART – B Short Answer **The answer should not exceed 200 words 5 x 4 = 20**		

11	a) Explain the closed operator with an illustration. (or) b) Explain the bounded operator with an illustration.	Explain Illustrate	Understand
12	a) Define Reflexive spaces and show that the space l^1 is not reflexive. (or) b) Define Hilbert spaces with proper examples.	Define Example	Understand
13	a) Describe Isolated singularities in detail. (or) b) Describe analytic function in detail.	Describe	Understand
14	a) Explain Cauchy Residue theorem and provide an application. (or) b) Illustrate an example for a Taylor series, Maclaurin series and Laurent's series	Illustrate	Apply
PART – C Essay Answer **The answer should not exceed 400 words 3 x 10 = 30**			
15	a) State and prove uniform boundedness principle. (or) b) State and prove closed graph theorem	Describe	Analyze
16	a) Explain the following i) Dual of $\mathbb{R}^n$ is $\mathbb{R}^n$. ii) Dual of $l^1 is l^\infty$. (or) b) Define the adjoint operator and its existence. Further supply an example using integral operator.	Explain Illustrate	Analyze
17	a) I) Let γ be a circle centred at 0 with radius 1. Using Residue theorem, evaluate the following i) $\int_\gamma \frac{\sin z}{z^4}\, dz$ ii) $\int_\gamma \frac{e^z}{z^3}\, dz$ II) Determine the constants a and b in order that the function $f(z) = (x^2 + ay^2 - 2xy) + (bx^2 - y^2 + 2xy)$ should be analytic. Further evaluate $f'(z)$. (or) b) I) Describe pole and residues of a function. II) Prove that the following functions are differentiable at every point (i.) f(z) = sin x cosh y + i cos x sinh y. (ii.) f(z) = (2x - 3y) + i(3x + 2y).	Evaluate	Skill

SEMESTER - II					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM022	Multivariate Statistical Analysis	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Select appropriate methods of multivariate data analysis, given multivariate data and study objectives;	**Remember**
CO 2	R programs to carry out multivariate data analyses, Understand likelihood based, as well as minimum expected cost based, discriminant analysis. Be able to apply these discriminant analysis methods to real data. Know how to display multivariate data graphically using R and be able to use the R package for multivariate data analysis.	**Understand**
CO 3	To select and master relevant statistical tools according to research questions and the formulated hypotheses. Know the theories of PCA and factor analysis (FA), and be able to apply these methods to real data.	**Apply**
CO 4	Interpret results of multivariate data analyses.	**Analyze**
CO 5	Understand the fundamental difference between univariate and multivariate analysis. Design/perform hypothesis testing (mainly the Hotelling T2 test and chi-square test) using multivariate data. Understand and be able to apply MANOVA and understand multivariate regression.	**Understand**

b. Syllabus

Units	**Content**	**Hrs.**
I	Reviews of Multivariate Distributions, Multiple and Partial Correlation, Multivariate Normal Distribution, Marginal and Conditional Distributions - Maximum likelihood Estimators of sample Mean and dispersion Matrix. Distribution of mean vector and Sample Dispersion Matrix - James-Stein Estimator for the Mean Vector, Wishart Distribution and its Properties (without derivation)	15
II	Tests based on Mean Vectors for one and two Multivariate Normal Distributions -Hotelling'sT2 and MahalanobisD2 test statistics with their null and non-null distributions - Related Confidence Regions - Testing and Illustration using likelihood Ratio Criterion**.**	15
III	Principal Component Analysis, Factor Analysis Underlying Models and Illustrations, Identification Problem, Estimation – Maximum likelihood Method, Centroid Method, Canonical Correlation – Extraction - Properties**.**	15
IV	Classification Analysis using Discriminate functions - Clustering techniques Hierarchical Clustering - Agglomerative techniques, Single Linkage Method, Complete average linkage method–non-hierarchical method–K-Mean**.**	15
	References: **1.** T.W. Anderson, An Introduction to Multivariate Statistical Analysis,	

	Second Edition, Wiley Eastern, 1980. 2. R. A. Johnson and D. W. Wichern, Applied Multivariate Statistical, 5th Edition, Upper Saddle River, NJ: Prentice Hall; 2002. 3. M. Jambu and M. O. Lebeaux, Cluster Analysis and Data Analysis, North Holland Publishing Company, 1983. 4. A. M.Kshirsagar, Multivariate Analysis, Marcel Decker.1972. 5. Härdle, W. K. & Simar, L. (2012). Applied Multivariate Statistical Analysis, Springer, New York 6. D. F. Morrison, Multivariate Statistical Methods, Second Edition, McGraw Hill, 1976.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	3	2	3	2
CO2	3	3	3	3	3
CO3	3	3	3	3	3
CO4	2	2	2	3	2
CO5	3	3	3	3	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Short Answer- 6 x 2 = 12 marks)**	4	4	2	2	2
Part – B **(Long Answer -4 x 12 = 48 marks)**	8	8	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

<table>
<tr><th></th><th>Model Questions</th><th>Specification</th><th>Level</th></tr>
<tr><td></td><td>Part – A: Short Answer (Attempt any SIX)
Marks: 6 x 2 = 12</td><td></td><td></td></tr>
<tr><td>1</td><td>Define the entropy of a Univariate Normal Distribution.</td><td>State</td><td>Analyze, Apply</td></tr>
<tr><td>2</td><td>How to represent the classifier using Network?</td><td>State</td><td>Remember,</td></tr>
<tr><td>3</td><td>Define the Maximum Likelihood estimation method.</td><td>Recall, State</td><td>Analyze, Apply,</td></tr>
<tr><td>4</td><td>Write the formula for Continuous Conditional distribution</td><td>Recall, State</td><td>Remember, Understand</td></tr>
<tr><td>5</td><td>What do you mean by the Hierarchical Clustering method?</td><td>Recall, State</td><td>Remember</td></tr>
<tr><td>6</td><td>Write the formula for Linear Regression</td><td>Recall</td><td>Analyze, Apply</td></tr>
<tr><td>7</td><td>What do you mean by Group Linkage in clustering?</td><td>Define</td><td>Remember</td></tr>
<tr><td>8</td><td>Write the concept of Linear Discriminant Analysis.</td><td>Recall, State</td><td>Apply, skill</td></tr>
<tr><td></td><td>PART – B Long Answer (Attempt any FOUR)
Marks: 4 x 12=48</td><td></td><td></td></tr>
<tr><td>9</td><td>a) In IPS selection, the following skills Intelligence, Assertiveness, and Age play a vital role in the design of skirts. The correlation between Intelligence and Assertiveness in a group of adults from 20 to 35 years old is 0.60. The correlation between Intelligence and age in the same group is 0.70 and the correlation between Assertiveness and age is 0.40. Obtain the Multiple correlations and write the inference.

b) The joint probability mass function of (X, Y) is given by
$p(x, y) = k(2x + 3y)$, $x = 0, 1, 2$, $y = 1, 2, 3$.
Find the value of k and Marginal probability distributions of X and Y.</td><td>Problem, Explain</td><td>Remember, Understand, Analyze, Apply, skill</td></tr>
<tr><td>10</td><td>a) Explain the different types of Cluster measures in multivariate data analysis
b) Obtain the covariance matrix for the given data
<table>
<tr><th>X</th><th>Y</th><th>Z</th></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
<tr><td>1</td><td>2</td><td>1</td></tr>
<tr><td>1</td><td>3</td><td>2</td></tr>
<tr><td>1</td><td>4</td><td>3</td></tr>
</table></td><td>Explain, Problem, Obtain</td><td>Remember, Understand, Analyze, Apply, skill</td></tr>
<tr><td>11</td><td>a) Compute the principal component by using PCA algorithm based on the feature vectors
(2,1); (3,5); (4,3); (5,6) ;(7,8) ;(10,5).

b) Explain the centroid method.</td><td>Problem, Explain</td><td>Remember, Understand, Analyze, Apply, skill</td></tr>
</table>

<table>
<tr>
<td>12</td>
<td>a) MRI has been taken for diagnosing the blood clot at KMC and the scores are given below.
<table>
<tr><td>True Positive =5</td><td>False Positive =30</td></tr>
<tr><td>False Negative=10</td><td>True Negative=400</td></tr>
</table>
Find the various measures of a classifier.

b) Explain canonical correlation and its properties.</td>
<td>Problem, Explain, obtain</td>
<td>Understand, Analyze, Apply, skill</td>
</tr>
<tr>
<td>13</td>
<td>a) Nissan company manufactures three different types of cars such as Magnet, Ready go and Kicks in Japan. Test whether there is any significant difference among the three varieties of cars manufactured by the company.
<table>
<tr><td>Magnet</td><td>20</td><td>21</td><td>23</td></tr>
<tr><td>Ready go</td><td>18</td><td>20</td><td>17</td></tr>
<tr><td>Kicks</td><td>25</td><td>28</td><td>22</td></tr>
</table>
(tabulated Value 5.14)
b) Use K means algorithm and divide the data into two clusters (2 iterations).
<table>
<tr><td>X1</td><td>1</td><td>2</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>X2</td><td>1</td><td>1</td><td>3</td><td>2</td><td>3</td><td>5</td></tr>
</table></td>
<td>Explain, Problem</td>
<td>Analyze, Apply, skill</td>
</tr>
<tr>
<td>14</td>
<td>Write short notes on any THREE of the following: (3 x 4 =12)
a) Explain confidence region in multivariate distribution.
b) Define distance measures and their properties.
c) Explain the concept of outlier with an example
d) How many dendrograms can be drawn with 15 leaves.
e) Explain dimension reduction techniques</td>
<td>Note</td>
<td>Remember, Understand, Analyze, Apply, skill</td>
</tr>
</table>

SEMESTER - II					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM023	Numerical Analysis (Theory and lab)	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Studysome direct and iterative methods for the solution of nonlinear equations, systems of linear equations, and nonlinear equations and also the error analysis and system conditions by using results from real analysis and linear algebra.	Remember
CO 2	Analyze the difference between numerical differentiation and integration.	Understand
CO 3	Understand polynomial interpolation, least square approximation, and spline interpolation to construct interpolating polynomials for given data, and also the difference between single-step and multi-step methods for initial-value ordinary differential equations.	Apply
CO 4	Explain the finite difference solution for a one-dimensional initial and boundary value ordinary and partial differential equations.	Analyze
CO 5	Formulate execution of numerical algorithms for direct and iterative methods as well as single-step and multi-step methods by using Scilab or MATLAB software.	Skill

b. Syllabus

Units	**Content**	**Hrs.**
I	**Linear and Nonlinear Equations:** Linear System (Direct methods); Gaussian elimination with pivoting and scaling; LU decomposition; Vector and matrix norms; Error analysis and system condition; Linear system (Iterative methods); Jacobi and Gauss-Seidel method; Convergence considerations; The Eigenvalue problems; Nonlinear equations; One-point iteration approach; Newton's method**.**	20
II	**Interpolation by Polynomials:** Lagrange's interpolation; Inverse interpolation; Accuracy of interpolation; Newton's divided differences interpolation; Errors in Lagrange's and Newton's divided differences interpolation; Relationship between derivatives and divided differences; Least squares approximation; Approximation by trigonometric polynomials; Fast Fourier transforms; Interpolation by splines**.**	15
III	**Numerical Differentiation and Integration:** Differentiation based on interpolation and divided differences; Newton-Cotes integration formulas (trapezoidal & Simpson rules); Gaussian quadrature; Error estimation in trapezoidal rule, Simpson rules, and Gaussian quadrature; Quadrature rules for multiple integrals**.**	10
IV	**Numerical Solution of Differential Equations: ODEs-** Single-point methods; Multipoint methods; Error estimation and convergence of the above methods; Finite difference methods; Consistency, order,	15

	stability, and convergence of numerical methods; **PDEs-** Steady-state two-dimensional (2D) Laplace equation; Finite difference solution of the Laplace equation; Unsteady 1D parabolic diffusion equation, Explicit and implicit schemes; Unsteady 1D convection hyperbolic equation; Explicit schemes for convection equation.	
	References: **1.** G. M. M. Phillips and P. J. Taylor, Theory and Applications of Numerical Analysis, Second Edition, Elsevier, 2006. **2.** E. Isaacson, H. B. Keller, Analysis of Numerical Methods, First Edition, Dover Publication, 1994. **3.** A.Quarteroni, F.Saleri and P.Gervasio, Scientific Computing with MATLAB and Octave, Springer, 2006. **4.** S. D. Conte and C. de Boor, Elementary Numerical Analysis: An Algorithmic Approach, Third Edition, McGraw-Hill, 1981.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	2	2	3	3
CO2	3	1	1	1	1
CO3	3	3	2	2	3
CO4	3	2	1	1	1
CO5	3	2	1	2	2

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 10 x 1 = 10 marks)**	2	2	2	2	2
Part – B **(Short Answer - 5 x 4 = 20 marks)**	10	10	-	-	-
Part – C **(Essay- 3 x 10 = 30 marks)**	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

Sl. No.	Model Questions	Specification	Level
	Part – A: Objective Type **Multiple Choice Marks: 10 x 1 = 10**		
1	Consider the number X which is rounded to N decimal places such that $\Delta X = \frac{1}{2}(10^{-N})$. If $X = 0.51$ and is correct to 2 decimal places, then its relative accuracy is given by (A) 0.98 % (B) 0.89 % (C) 9.8 % (D) 98 %	Find	Apply, Skill
2	If $x = (1, -2, 3), y = (0, 2, 3)$ are two vectors in $\mathbb{R}^3$, then find $\lVert x + y \rVert_\infty$. (A) 6 (B) 1 (C) 3 (D) 0	Find	Apply, Skill
3	Which of the following relation is true? (A) $\Delta \equiv E - 1$, (B) $E^n \equiv (1 + \Delta)^n$, (C) both A and B, (D) None of these	Recall, Check	Remember, Understand, Analyze
4	Find the interpolating polynomial for a function f, given that $f(x) = 0, -3$ when $x = 1, -1$ (A) $\frac{3}{2}(x - 1)$ (B) $\frac{3}{2}(1 - x)$ (C) $\frac{1}{2}(x - 1)$ (D) $\frac{1}{2}(1 - x)$	Find	Apply, Skill
5	If $f(x)$ is a polynomial of degree n in x, then nth difference of this polynomial is (A) Constant (B) Variable (C) Zero (D) None of these	Recall	Analyze, Understand
6	The numerical value obtained by applying the two-point trapezoidal rule to the integral $\int_0^1 \frac{\ln(1+x)}{x} dx$ is (A) $\frac{1}{2}(\ln 2 - 1)$ (B) $\frac{1}{2}$ (C) $\frac{1}{2}\ln 2$ (D) $\frac{1}{2}(\ln 2 + 1)$	Evaluate	Analyze, Apply, Skill
7	In Euler's method: Given initial value problem (IVP), $y' = \frac{dy}{dx} = f(x, y)$, with $y(x_0) = y_0$, then approximation is given by (A) $y_{n+1} = y_n + hf(x_{n-1}, y_{n-1})$, where $h = (x_n - x_0)/n$ (B) $y_{n+1} = y_n$, (C) $y_{n+1} = y_n + hf(x_n, y_n)$, where $h = (x_n - x_0)/n$ (D) None of these	Recall,	Remember, Understand
8	If X_0 is an approximation to the solution of $\begin{bmatrix} 1.00 & 0.500 & 0.333 \\ 0.500 & 0.333 & 0.250 \\ 0.333 & 0.250 & 0.200 \end{bmatrix} X = \begin{bmatrix} 1.83 \\ 1.08 \\ 0.783 \end{bmatrix}$, then find the residual vector r_0 corresponding to X_0 where $X_0 = [1\ 1\ 1]^T$ (A) $[0.003, 0.003, 0]^T$,B)$[0.03, 0.03, 0]^T$,C)$[0.003, -0.003, 0]^T$,D)$[0.03, -0.03, 0]^T$	Find	Apply, Skill, Analyze
9	Given y_0, y_1, y_2, y_3 corresponding to the values x_0, x_1, x_2, x_3 for the function $y = f(x), a \le x \le b$. Let $f(x)$ is a polynomial of degree 3. Then by Simpson's $\frac{3}{8}$rule, the integral $I = \int_a^b f(x)\, dx$ is equivalent to (A) $I = \frac{3}{8}h[y_0 + 3y_1 + 3y_2 + y_3]$, (B) $I = \frac{1}{3}h[y_0 + 4y_1 + y_2]$ (C) $I = \frac{1}{3}h[2y_0 + 4y_1 + 2y_2]$, (D) $I = \frac{1}{2}h[y_0 + 2y_1 + 2y_2 + y_3]$	Recall	Remember, Understand
10	Which of the following is the 2nd-order centered space approximation of the firstorder spatial derivative $\overline{f_x}$.	Check, State	Analyze, Understand

(A) $\frac{f_{i+1}-f_i}{\Delta x}$ B) $\frac{f_i-f_{i-1}}{\Delta x}$ C) $\frac{f_{i+1}-f_{i-1}}{2\Delta x}$ D) $\frac{f_{i+1}+f_{i-1}}{2\Delta x}$

PART – B Short Answer

The answer should not exceed 200 words Marks:5 x 4 = 20

11

a) Ava invests a total of \$10,000 in three accounts. One paying 5% interest; another paying 8% interest, and the third paying 9% interest. The annual interest earned on the three investments last year was \$770. The amount invested at 9% was twice the amount invested at 5%. How much was invested at each rate? (Hint: Use Gauss elimination method).

(or)

b) The upward velocity of a rocket is given as a function of time in the following table. Find the velocities at $t = 16$ and 18 seconds using linear splines.

Table: Velocity as a function of time

$t(s)$	0	10	15	20	22.5	30
$v(t)\ (m/s)$	0	227.04	362.78	517.35	602.97	901.67

Examine, Find, Solve, Formulate, | Understand, Analyze, Apply, Skill

12

a) Construct a six-place divided difference table for the function $f(x) = \frac{1}{x}$ for $3.1 \le x \le 3.9$ with $\Delta x = 0.1$.

(or)

b) Find the value of $f(x) = \log x$ at $x = 2.14$using the Lagrange interpolation, given that $f(x) = 0.7419, 0.7885$ when $x = 2.1, 2.2$ respetively. Also, show that the error estimation in the finding the interpolating polynomial $'f'$ is lying between 0.00024 and 0.00028(Correct to **5** decimal places).

Find, Check, Verify, Recall | Remember, Analyze, Apply, Skill

13

a) Find the first and second derivative of the function $f(x)$ at the point $x = 1$ for the following tabulated data:

x	1	2	3	4	5	6
$y = f(x)$	1	8	27	64	125	216

(or)

b) What are the Gaussian quadrature rules? Also, derive the Gaussian quadrature rule with 2 points over the inetrval [-1, 1].

Find, Define, Derive, Recall | Remember, Understand, Skill

14

c) Evaluate $\int_0^{\pi/2}\int_0^{\pi/2}\sqrt{\sin(x+y)}\,dxdy$ by numerical double integration (the interval can be divided into four equal subintervals and correct to **4** decimal places).

(or)

d) Solve the initial value problem (IVP) $\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$ from $x = 0$ to $x = 1$, and the initial condition at $x = 0$ is $y = 1$ by 4th order Runge-Kutta method with step size $h = 0.5$(Carry out **7** decimal places in calculations).

Evaluate, Solve, Check, Examine | Understand, Analyze, Apply, Skill

PART – C Essay Answer

The answer should not exceed 400 words Marks: 3 x 10 = 30

15 **a)** If $A = [a_{ij}]_{n\times n}$ is a matrix such that all $'n'$ of its leading sub-matrices are | Understand,

	non-singular, then prove that there exists a unique factorization of the form, $A = LDV$ where $'L'$ is lower triangular with units on the diagonal, $'V'$ is upper triangular with units on the diagonal, and $'D'$ is diagonal with non-zero diagonal elements. **(or)** **b)** Compute the three iterations of the Gauss-Seidel method by solving the following system of equations, starting with an initial vector $X_0 = [0 \quad 0 \quad 0]^T$, and working to **3** decimal places. $4.63x_1 - 1.21x_2 + 3.22x_3 = 2.22,$ $-3.07x_1 + 5.48x_2 + 2.11x_3 = -3.17,$ $1.26x_1 + 3.11x_2 + 4.57x_3 = 5.11,$	Recall, State, Define, Prove, Compute	Analyze, Apply, Skill
16	**a)** Given the following data, use the Newton forward difference interpolation formula to estimate $\sin x$ at $x = 0.63, 0.65$ and determine the accuracy of the result (Correct to **6** decimal places). x: 0.6, 0.7, 0.8, 0.9, 1.0 $\sin x$: 0.564642, 0.644218, 0.717356, 0.783327, 0.841471 **(or)** **b)** Find the integral of exp $(-x^2)$ between $x = 0.2$ and $x = 2.6$. Compare the results at varying values for the subdivisions $N = 6, 12, 18$ with the trapezoidal rule and Simpson's 1/3 rule (carry out 5 significant digits in the calculations).	Find, Recall, Examine, Verify	Remember, Understand, Analyze, Skill
17	**a)** Employ the 4th order Adams-Bashforth method to find $y(1.5)$, $y(2.0)$ for the differential equation $\frac{dy}{dx} = -y^2, y(1) = 1$, with $h = 0.1$ over the interval $1 \le x \le 2$, and the first four starting values are obtained from the exact solution of given differential equation (Carry out **8** decimal places in calculations). **(or)** **b)** Perform von Neumann stability analysis of the forward-time-centered-space (FTCS) method of the diffusion equation, $k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$.	Perform, Find, Examine	Remember, Understand, Analyze, Skill, Apply

x	0.6	0.7	0.8	0.9	1.0
$\sin x$	0.564642	0.644218	0.717356	0.783327	0.841471

SEMESTER - II					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM024	Differential Equations	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Explain the basics of Ordinary Differential Equations (ODEs), Partial Differential Equations (PDEs) along with their classifications.	**Remember**
CO 2	Understand the existence and uniqueness of solution for first order ODEs and solving procedures of first and second order ODE	**Understand**
CO 3	Apply a range of techniques to solve PDEs.	**Apply**
CO 4	Analyze the solution nature of ODEs, PDEs.	**Analyze**
CO 5	Demonstrate the modelling of physical phenomena using ODEs and PDEs.	**Skill**

b. Syllabus

Units	**Content**	**Hrs.**
I	First order linear ordinary differential equation (ODE)-The method of successive approximations, Lipchitz condition, Convergence of successive approximations, Existence and Uniqueness of solutions for first order initial value problem**.**	15
II	Second order linear ODE - General solution of homogeneous equations, non-homogeneous equations, Wronskian, Method of variation of parameters, Boundary value problems, Green's functions, Sturm-Liouville problems**.**	15
III	First order partial differential equation (PDE), Quasi linear PDE of the first order, Integral surfaces passing through a given curve, Surfaces orthogonal to the given system, Classification of integrals, Compatible systems of first order PDE, Charpit's method, Method of Characteristics, Nonlinear partial differential equation for first order**.**	15
IV	Second order PDE- Origin and Classification, linear second and higher order PDE with constant and variable coefficients, Characteristics curve of the second order PDE, Canonical form, Monge's method of solution of non-linear second order PDE**.**	15
	References: **1.** M**.** Braun, Differential Equations and their Applications, Fourth Edition, Springer, 1993**.** **2.** E**.** A**.** Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India Ltd., 2002**.** **3.** G.F**.** Simmons, Differential Equations with Applications and Historical Notes, Tata McGraw Hill, 2003**.** **4.** F**.** John, Partial Differential Equations, second edition, Springer-Verlag, 1978**.** **5.** L.C**. Evans, Partial Differential Equations, AMS, 2010.**	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	2	2	1	1
CO2	3	2	2	1	2
CO3	3	2	2	2	3
CO4	3	2	2	2	3
CO5	3	2	2	2	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A (Objective - 10 x 1 = 10 marks)	2	2	2	2	2
Part – B (Short Answer - 5 x 4 = 20 marks)	10	10	-	-	-
Part – C (Essay- 3 x 10 = 30 marks)	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

Sl. No.	Model Questions	Specification	Level
	Part – A: Objective Type **Multiple choice 10 x 1 = 10**		
1	Identify the solution of the differential equation $(y')^2 + y" - y = (\cos x)^2 + 2\sin x$. A. $\sin x$ B. $\cos x$ C. $\csc x$ D. $\cot x$	Identify	Remember
2	Identity the incorrect choice. A. The differential equation exists for ``all straight line passing through the origin". B. The function $z = e^x \cos y$ is a solution of $z_{xx} + z_{yy} = 0$ on the domain ℝ. C. The solution of the differential equation $y' + ay = 0$ decays to zero for $a < 0$ and grows to ∞ for $a > 0$. D. The solution of the differential equation $y' - ay = 0$ decays to zero for $a < 0$ and grows to ∞ for $a > 0$.	Identify	Remember
3	State the largest interval on which a solution exists for the initial value problem $(x^2 - 1)y'' - xy' + y = 0,;\ y(2) = a;\ \ y'(2) = b.$ A. $(1, \infty)$ B. $(0, \infty)$ C. $(-\infty, -1)$ D. $(-\infty, 1)$	Identify	Remember
4	Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions $of\ y'' + p(x)y' + q(x)y = 0$. Then find the condition in which $\varphi(x) = ay_1(x) + by_2(x)$ and $\psi(x) = cy_1(x) + dy_2(x)$ are linearly independent solutions of the same equation. $A. ad - bc = 0$ B. $ad - bc \neq 0$ C. $ab - cd = 0$ D. $ab - cd \neq 0$	Identify	Remember
5	Identify the sentences whether it is true or false. i) If f(x) and g(x) are linearly independent functions on an interval I, then they are linearly independent on any smaller interval contained in I. ii) If $y_1(x)$ and $y_2(x)$ are linearly dependent solutions of $y" + p(x)\, y' + q(x)\, y = 0$ on an interval I, then they are linearly dependent on any interval contained in I. A. i) and ii) are true B. i) and ii) are false C. i) is true and ii) is false D. i) is false and ii) is true	Recognize	Remember
6	Solve and pick the correct answer. The characteristic curve for the PDE xp-yq=z in XY plane is A. straight line with slope 1. B. straight line with slope -1. C. circle with center (0,0). D. circle with center (1,1).	Recognize	Remember
7	Evaluate $z(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, if z(x,y) is solution of the equation $xz_y - yz_x = 0$ with z(x,0) =sin$\frac{\pi x}{4}$. A.$\frac{1}{\sqrt{2}}$ B. $\frac{1}{2}$C$\frac{\sqrt{3}}{2}$D. 1.	Evaluate	Remember
8	Pick the number of solutions of the Cauchy problem $z_x + z_y = z$ with initial curve $(s, s, \sin s)$, $0 \leq s \leq 1$. A. 1 B. 0 C. 2 D. ∞	Recognize	Remember

9	Determine the category of the second order PDE $$z_{xx} + 2\cos x\, z_{xy} - sin^2 x\, z_{yy} = 0.$$ A. Hyperbolic B. Parabolic C. Elliptic D. None of the above	Recognize	Remember
10	Eliminate the constants and identify the first order PDE $z = ax + \frac{y}{a} + b.$ A. $pq = 1$ B. $p = q$ C. $p^2 + q^2 = 2$ d. $p^2 + q^2 = 1$	Identify	Remember
	PART – B Short Answer **The answer should not exceed 200 words 5 x 4 = 20**		
11	a) The differential equation $f(t)y'(t) + t^2 + y = 0$ is known to have an integrating factor $(t) = t$. Find all the possible function $f(t)$. (or) b) Find the values of x for which the solution $y(x)$ of the following initial value problem (IVP) exists. $$yy' + (1 + y^2)\sin x = 0$$ $$y(0) = 1$$	Discuss	Understand
12	a) Discuss the Wronskian matrix and relate it with the linear dependency of functions. (or) b) Solve the second order differential equation $(1 - x^2)y'' - 2xy' + 2y = 0$ with one known solution $y_1(x) = x$ as a solution.	Discuss	Understand
13	a) Discuss two real-life examples for ODE. (or) b) Discuss two real-life examples for PDE.	Demonstrate	Skill
14	a) Solve the equation$r = 4t$ by Monge's method. (or) b) Verify that $xp - yq - x = 0; x^2 p + q - xz = 0$are compatible systems. If yes, state the domain in which they are compatible. Also find the common solution.	Discuss	Apply
	PART – C Essay Answer **The answer should not exceed 400 words 3 x 10 = 30**		
15	a) Explain the Picard's successive approximation and the existence and uniqueness theorem (or) b) Examine the exactness of the differential equation and solve it. $(3y + e^x) + (3x + \cos y)\frac{dy}{dx} = 0$	Explain Identify	Analyse
16	a) Explain the strum Liouville boundary value problem and the characteristic values and functions (or) b) Find the surface which is orthogonal to the one parameter system $z(x + y) = c(3z + 1)$ and which passes through the circle $x^2 + y^2 = 1,\ z = 0.$.	Explain Discuss	Apply
17	a) Solve the water tank problem to find the height of water. (or) b) Explain the method of characteristics and solve the transport equation.	Demonstrate	Skill

SEMESTER - II					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM025	Statistical Inference	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

(Course outcomes are specific for a particular course. CO should be specific, measurable, achievable, realistic, and time-bound)

	Course Outcome	Level
CO 1	Define basic terms in frequentists approach and Bayesian approach to Statistical inference.	**Remember**
CO 2	Explain the concept of estimation, testing of hypothesis and confidence intervals.	**Understand**
CO 3	Derive estimators, tests and confidence intervals using Rao-Blackwell theorem, Lehmann-ScheffeTheorem, Bayes theorem, Neyman-Pearson Lemma, etc. Apply the inference procedures on data.	**Apply**
CO 4	Compare estimators and tests.	**Analyze**
CO 5	Interpret the results of Statistical Inference procedures.	**Skill**

b. Syllabus

Units	Content	Hrs.
I	Sufficiency principle, factorization theorem, minimal sufficiency, minimal sufficient partition, minimal sufficient statistics, minimal sufficient statistic for exponential family, power series family, curved exponential family, and Pitman family, completeness, bounded completeness, ancillary statistics, Basu’s theorem and its applications, conditionality principle	15
II	Problem of point estimation, unbiased estimators, minimum variance unbiased estimator, Rao-Blackwell theorem and Lehmann-Scheffe theorem and their applications. A necessary and sufficient condition for an estimator to be UMVUE, Fisher information and information matrix, Cramer-Rao inequality. Maximum likelihood estimator (MLE), properties of MLE, MLE in nonregular families, method of scoring and its applications**.**	15
III	The concepts of prior and posterior distributions, conjugate, Jeffrey’s and improper priors with examples, Bayes estimation under squared error and absolute error loss functions**.**	15
IV	Problem of testing of Hypothesis, Simple and composite hypotheses. Randomized and non-randomized tests, Most powerful test, Neyman-Pearson Lemma and its applications. Monotone likelihood ratio property, UMP test, power function of a test, existence of UMP test, UMP test for one-sided alternatives. Concept of p-value. Problem of confidence intervals, relation with testing of hypotheses problem, UMA confidence intervals, shortest length confidence intervals. Likelihood ratio test and its applications. Commonly used statistical tests**.**	15
	References:	

	1. Rohatgi, V.K. and Saleh, A. K. MD. E. (2015). Introduction to Probability Theory and Mathematical Statistics -3rd edition, John Wiley & sons. 2. Lehmann, E. L. (1983). Theory of Point Estimation - John Wiley & sons. 3. Rao, C. R. (1973). Linear Statistical Inference and its Applications, 2nd edition, Wiley. 4. Kale, B.K. and Muralidharan, K. (2015). Parametric Inference: An Introduction, Alpha Science International Ltd. 5. Mukhopadhyay, P. (2015). Mathematical Statistics, Books and Allied (p) Ltd. 6. Dudewicz, E. J. andMishra,S. N. (1988). Modern Mathematical Statistics, John Wiley and Sons. 7. Casella, G. and Berger, R. L. (2001). Statistical Inference, 2nd edition, Duxbury press.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	2	1	1	1
CO2	2	3	2	1	2
CO3	2	3	2	3	3
CO4	2	3	2	1	3
CO5	1	2	3	1	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 6 x 2 = 12marks)**	4	4	2	2	2
Part – B **(Short Answer - 4 x 12 = 48marks)**	8	8	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

g. Rubric for Assignments

Sl. No.	Criteria	100%	75%	50%	25%	0%	Relation to COs
1	**Content** **50%**	Ideas are detailed, well developed, supported with specific evidence &facts andexamples	Ideasaredetailed, Developed and supported with evidence and facts mostly specific.	Ideas are presented but not particularly developed or supported;	Content is not sound	Not attended	CO1, CO2, CO5
2	**Organiza-tion** **50%**	Includes title, introduction,statement ofthemain idea withillustration and conclusion.	Includes title, introduction,statementofmainideaand conclusion.	organizational tools are weak or missing	Noorganization	Not attended	CO1, CO2, CO5

h. Rubric for Seminar

Sl. No.	Criteria	100%	75%	50%	25%	0%	Relation to COs
1	**Knowledge and Understanding** **50%**	Exceptional knowledge of facts, terms, and concepts	Detailed knowledge of facts, terms, and concepts	Considerable knowledge of facts, terms, and concepts	Minimal knowledge of facts, terms, and concepts	Not Attended	CO3, CO4
2	**Presentation** **50%**	Well Communicated with logical sequences, examples, and references	Communicated with sequences	Just Communicated	No coherent communication	Not Attended	CO3, CO4

i. Model Question Paper

<table>
<tr><th>Sl. No.</th><th>Model Questions</th><th>Specification</th><th>Level</th></tr>
<tr><td></td><td>Part – A: Short Answer (Attempt any SIX)
Marks: 6 x 2 = 12</td><td></td><td></td></tr>
<tr><td>1</td><td>State Basu’s theorem. Give its application.</td><td>Recall</td><td>Remember</td></tr>
<tr><td>2</td><td>Describe minimal sufficient statistic. Give an example.</td><td>Recall, Example</td><td>Remember, Understand</td></tr>
<tr><td>3</td><td>Define unbiased estimator. Give an example.</td><td>Recall, Example</td><td>Remember, Understand</td></tr>
<tr><td>4</td><td>Define sufficient statistic. Give an example.</td><td>Recall, Example</td><td>Remember, Understand</td></tr>
<tr><td>5</td><td>What is prior distribution?</td><td>Recall</td><td>Remember</td></tr>
<tr><td>6</td><td>What are simple and composite hypothesis?</td><td>Recall</td><td>Remember</td></tr>
<tr><td>7</td><td>Is most powerful test unique? Justify.</td><td>Example</td><td>Understand</td></tr>
<tr><td>8</td><td>State the role of Type I and Type II error in testing of hypothesis.</td><td>Role</td><td>Analyze</td></tr>
<tr><td></td><td>PART – B Long Answer (Attempt any FOUR)
Marks: 4 x 12=48</td><td></td><td></td></tr>
<tr><td>9</td><td>a) State and Prove Rao-Blackwell theorem.
b) Define UMVUE. Let $X_1, X_2, \dots, X_n$ be a random sample from $U(0,\theta)$. Find UMVUE of θ.</td><td>State, Prove, Define, Problem</td><td>Remember, Understand, Analyze, Apply</td></tr>
<tr><td>10</td><td>a) State Neyman Factorization theorem. Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\theta,1)$. Find sufficient statistic for θ.
b) Explain the following terms:
i) Exponential family of distributions
ii) Pitman family of distributions
iii) Power series family of distributions</td><td>State, Prove, Define, Problem</td><td>Remember, Understand, Analyze, Apply</td></tr>
<tr><td>11</td><td>a) Define maximum likelihood estimator. Let $X_1, X_2, \dots, X_n$ be a random sample from $Poisson(\theta)$. Find maximum likelihood estimator of θ.
b) Explain the following terms:
(i) Conjugate Prior, (ii) Jeffery’s Prior</td><td>State, Prove, Define, Problem</td><td>Remember, Understand, Analyze, Apply</td></tr>
<tr><td>12</td><td>a) State Neyman-Pearson Lemma. Find MP test of size $\alpha = 0.05$ based on a single observation for testing $H_0: X \sim P_0$ Vs $H_1: X \sim P_1$, where P_0 and P_1 are as follows:
<table>
<tr><th>X</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><td>P_0</td><td>0.05</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.35</td></tr>
<tr><td>P_1</td><td>0.35</td><td>0.3</td><td>0.2</td><td>0.1</td><td>0.05</td></tr>
</table>
b) What is monotone likelihood ratio property? Does $N(\theta,1)$ satisfy monotone likelihood ratio property? Justify.</td><td>State, Prove, Define, Problem</td><td>Remember, Understand, Analyze, Apply</td></tr>
<tr><td>13</td><td>a) Define Likelihood ratio test. Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\theta,1)$. Find Likelihood ratio test of size α for testing $H_0: \theta = \theta_0$ Vs $H_1: \theta \neq \theta_0$.
b) Explain the terms:</td><td>State, Prove, Define, Problem</td><td>Remember, Understand, Analyze, Apply</td></tr>
</table>

	(i) UMA confidence interval, (ii) Shortest length confidence interval.		
14	Write short notes on any **THREE** of the following: (3 x 4 =12) a) Cramer-Rao lower bound b) Complete statistic c) Baye's Estimator under squared error loss d) Uniformly most powerful test e) Fisher Information and Information matrix	Note	Remember, Understand, Analyze, Apply

SEMESTER - III					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM031	Fluid Dynamics	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	Level
CO 1	Understand the basic concepts of fluid dynamics and methods of describing fluid motion.	**Remember**
CO 2	Analyze the stress-strain rate relationship in fluid flow.	**Understand**
CO 3	Apply Bernoulli's principle, conservation of mass, conservation of momentum, and conservation of energy equations to solve some simple fluid flow problems.	**Apply**
CO 4	Explain Blasius and Kutta-Joukowski's theorems and their applications.	**Analyze**
CO 5	Illustrate dimensional analysis, the law of similarity, and boundary layer theory to solve fluid flow problems.	**Skill**

b. Syllabus

Units	Content	Hrs.
I	Continuum hypothesis, Forces acting on a fluid; Analysis of the relative motion near a point; Transport theorem; Methods of describing fluid motion; Translation, rotation and rate of deformation; Differentiation following the motion of the fluid; Classification of fluids; Conservation laws; Equation of continuity; Euler's equation; Equations of motion (Navier-Stokes equations); Energy equation.	15
II	Streamlines; Equation of state (**EOS**); Isentropic fluids; Vorticity; Theory of stress and rate of strain; Relationship between them; Kelvin's circulation theorem; Helmholtz's theorem; Rotational and irrotational flows; Bernoulli's equation; Momentum theorem and its applications; Two dimensional irrotational flow of an incompressible fluid.	15
III	Stokes' stream function; Axisymmetric flows; Gravity waves; Damping of gravity waves; Flow in a pipe; Potential flow; Complex potential, Blasius theorem; Kutta-Joukowski theorem; D'Alembert paradox.	15
IV	Dimensional analysis; Law of similarity and the Reynolds number; Flow between two parallel flat plates; Couette flow; Poiseuille flow; Torque and drag on a sphere due to a uniform flow; Flow with small Reynolds numbers; Stoke's law; Unsteady motion of a flat plate; Boundary layers; Prandtl's boundary layer equations; Solution for steady flow on a flat plate of infinite length.	15
	References: **1.** G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, First Edition, 1993. **2.** A. J.Chorin and J. E. Marsden, A Mathematical Introduction to Fluid Mechanics (Texts in Applied Mathematics), Third	

	Edition, Springer-Verlag, 1993. **3.** L. D. Landau and E. M.Lifshitz, Fluid Mechanics (Volume 6 of Course of Theoretical Physics), Pergamon Press, Second Edition, 1987. **4.** M. E. O'Neill and F. Chorlton, Ideal and Incompressible Fluid Dynamics (Mathematics and Its Applications), Ellis Horwood Ltd, Publisher, 1986.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	3	1	0	0
CO2	3	2	2	0	1
CO3	3	3	1	1	1
CO4	3	2	2	1	0
CO5	3	1	1	2	0

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 10 x 1 = 10 marks)**	2	2	2	2	2
Part – B **(Short Answer - 5 x 4 = 20 marks)**	10	10	-	-	-
Part – C **(Essay- 3 x 10 = 30 marks)**	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

Sl. No.	Model Questions	Specification	Level
	Part – A: Objective Type **Multiple Choice Marks: 10 x 1 = 10**		
1	Which of the following is the basic principle of fluid mechanics? (A) Continuity equation (B) Momentum principle (C) Energy equation (D) All of the mentioned	Recall	Remember
2	The dynamic viscosity of a liquid is $1.2 \times 10^{-4} Ns/m^2$, whereas, the density is 600 kg/m^3. The kinematic viscosity in m^2/s is (A) 72×10^{-3} (B) 7.2×10^3 (C) 20×10^{-8} (D) 70×10^6	Find	Apply, Skill
3	The continuum assumption of fluid mechanics no longer holds when the (A) Prandtl number is large (B) Knudsen number is large (C) Reynold's number is unity (D) Prandtl number is small	Explain	Remember, Analyze
4	The pressure of a water in a pipe when water is not flowing is 3×10^5 Pa when the water flows the pressure falls 2.5×10^5 Pa. Find the speed of flow of water (in m/s)? (A) 1 (B) 5 (C) 10 (D) 20	Find	Apply, skill
5	What is D'Alembert's paradox? (A) Resistance = 0 (B) Drag force = 0 (C) Temperature = 0 (D) Pressure Gradient = 0	Recall, State	Remember, Understand
6	The shearing stress in a piece of structural steel is 100 MPa. If the elastic modulus is 200 GPa and the Poisson's ration is 0.25, then the shearing strain γ would be (A) 800 (B) 1.25 (C) 0.8 rad (D) 0.00125 rad	Find	Apply, skill
7	Which of these conditions is not met a point for irrotational flow? (A) $\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$ (B) $\frac{\partial w}{\partial x} = \frac{\partial u}{\partial z}$ (C) $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial x}$ (D) $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial z}$	Check, Verify	Analyze, Apply
8	Which of the following is the mathematical technique used to predict physical parameters? (A) Dimensional analysis (B) Pressure analysis (C) Combustion analysis (D) Temperature analysis	Check	Aanlyze, Understand
9	If x is the distance measured from the leading edge of a flat plate, the laminar boundary layer thickness varies as (A) $\frac{1}{x}$ (B) $x^{4/5}$ (C) x^2 (D) $x^{1/2}$	Find	Apply, skill
10	The velocity corresponding to Reynolds number of 2800 is called as (A) Lower critical velocity (B) Subsonic velocity (B) Supersonic velocity (D) Hyper critical velocity	Recall	Remember, understand
	PART – B Short Answer **The answer should not exceed 200 words Marks:5 x 4 = 20**		
11	**a)** Consider an incompressible steady flow with constant viscosity. The velocity component is given by $u(y) = y\frac{U}{h} + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\frac{y}{h}\left(1 -\right.$	Find, Problem,	Analyze, Apply, skill

	$\frac{y}{h}\big)$ and $v = w = 0$ where h,U, and dp/dx are constants, and $p = p(x)$. If the body force is neglected, then does $u(y)$ satisfy the equation of motion? **(or)** **b)** Derive the Bernoulli equation when the compressibility effects are not negligible for an ideal gas undergoing **i)** an isothermal process and **ii)** an isentropic process.	Derive	
12	**a)** If $\boldsymbol{p}$ denotes the pressure, $\boldsymbol{V}$ be the potential of the external forces and $\boldsymbol{q}$ be the velocity of a homogeneous liquid moving irrotationally, then show that $\nabla^2 q^2$ is positive and $\nabla^2 p$ is negative provided $\nabla^2 V = 0$. **(or)** **b)** Consider the flow field given by $\psi = a(x^2 - y^2)$, where $a = 3s^{-1}$ **(i)** determine whether the flow is irrotational or not. **(ii)** find the velocity potential (ϕ) for this flow. **(iii)** show that lines of constant ϕ are orthogonal to lines of constant ψ.	Problem, Find, Prove	Understand, Analyze, Skill
13	**a)** Define potential and complex potential flows. An irrotational two-dimensional flow has stream function $\psi = A(x - c)y$, where A and c are constants. If the radius of the circular cylinder is a, and its centre being at the origin, then find the complex potential of the resulting flow. **(or)** **b)** What is the gravity wave in water? How do the individual elements of fluid move in a gravity wave?	Define, Explain, Find, Problem	Understand, Analyze, Recall, Skill
14	**a)** What restrictions or conditions are imposed on stream function ψso that it exactly satisfies the two-dimensional incompressible continuity equation by definition? Why are these restrictions necessary? **(or)** **b)** The drag force F, on a smooth sphere, depends on the size of the sphere (characterized by the diameter D), the relative velocity V, the fluid density ρ, and the fluid viscosity μ. It can be represented by the symbolic equation $F = f(\rho, V, D, \mu)$. Obtain the set of dimensionless groups by employing the method or repeating variables [Take ρ, V, D are repeating parameters].	Examine, Recall, Obtain, Problem	Analyze, Apply, skill
	PART – C Essay Answer **The answer should not exceed 400 words Marks: 3 x 10 = 30**		
15	**a)** If the velocity components of fluid particle at a point $P(x, y, z)$ are given by $u = cx + 2\omega_0 y + u_0$,$v = cy + v_0$,$w = -2cz + w_0$ where c,ω_0,u_0,v_0, and w_0 are constants. Then determine the velocity components at a neighboring point $Q(x_1, y_1, z_1)$ and determine the different types of motion which are involved. **(or)** **b)** State and derive the law of conservation of mass.	Sate, Problem, Prove	Remember, Understand, Analyze, Apply, skill
16	**a)** State and prove Kelvin's circulation theorem. **(or)**	State, Prove	Remember, Understand,

	b) State and prove Kutta-Joukowski lift theorem**.**		Analyze
17	**a)** If $w(z) = U\left(z + \frac{a^2}{z}\right) - \frac{i\Gamma}{2\pi}\log z$, (where z is the complex variable and Γ is an arbitrary strength of the line vortex flow) is the complex potential of a more general irrotational flow having no normal velocity at $\lvert z \rvert = a$, yet being uniform flow past a circular cylinder with speed U, at infinity, then apply Blasius's theorem to calculate lift and drag force on the cylinder. Also, find the moment of the resultant force**.** **(or)** **b)** Derive exact solution of Navier-Stokes equations for an unsteady motion due to impulsive acceleration of an infinite flat plate in a viscous incompressible fluid**.**	Problem, Prove, Find, Sate	Remember, Understand, Analyze, Apply, skill

SEMESTER - III					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM032	Stochastic Processes	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	Level
CO 1	Understand the axiomatic formulation of modern Probability Theory and think of random variables as an intrinsic need for the analysis of random phenomena.	**Remember**
CO 2	Characterize probability models and function of random variables based on single & multiples random variables.	**Understand**
CO 3	Evaluate and apply moments & characteristic functions and understand the concept of inequalities and probabilistic limits.	**Apply**
CO 4	Understand the concept of random processes and determine covariance and spectral density of stationary random processes.	**Analyze**
CO 5	Demonstrate the specific applications to Poisson and Gaussian processes and representation of low pass and band pass noise models	**Skill**

b. Syllabus

Units	Content	Hrs.
I	Stochastic processes and their classification-Brownian Motion–Markov chain– Examples (Random walk, Gambler's ruin problem)- classification of states of a Markov Chain-Recurrence-Basic limit theorem of Markov Chains-Absorption probabilities and criteria for recurrence.	20
II	Markov chains continuous in time – General pure birth processes and Poisson process, birth and death processes, finite state continuous time Markov chains.	15
III	Branching processes discrete in time – Generating functions relations – Mean and variance – Extinction probabilities – Concept of Age dependent Branching process.	15
IV	Renewal processes – Definition and examples – key renewal theorem – Study of residual life time process.	10
	References: **1.** S. Karlin, and H.M. Taylor, A First Course in Stochastic Processes, Academic Press, 1975. **2.** J. Medhi, Stochastic Processes, 3rd Edition, New age International, 2009. **3.** B.R. Bhat, Stochastic Models: Analysis and Applications, New Age Publications, 2004. **4.** P.W. Jones, and P. Smith, Stochastic Processes: An Introduction, Arnold Press, 2001. **5.** E. Cinlar, Introduction to Stochastic Processes, Prentice-Hall Inc., 1975.	

	6. D. R. Cox, and H.D. Miller, Theory of Stochastic Processes, 3rd Edition, Chapman and Hall, 1983. 7. S.M. Ross, Stochastic Process, Wiley, 1983.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	3	3	3	2
CO2	3	3	3	3	3
CO3	3	3	3	3	3
CO4	2	2	1	3	2
CO5	1	1	1	1	2

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 6 x 2 = 12 marks)**	4	4	2	2	2
Part – B **(Short Answer - 4 x 12 = 48 marks)**	8	8	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

	Model Questions	Specification	Level
	Part – A: Short Answer (Attempt any SIX) **Marks: 6 x 2 = 12**		
1	Define Stochastic Processes	State	Analyze, Apply
2	Define Markov Chain.	State	Remember,
3	What do you mean by Random Walk	Recall, State	Analyze, Apply,
4	Define finite state continuous time Markov chains.	Recall, State	Remember, Understand
5	List the classification of Stochastic processes	Recall, State	Remember
6	Give an example for extinction probability?	Recall	Analyze, Apply
7	Define Renewal Processes.	Define	Remember
8	State Key Renewal Theorem	Recall, State	Apply, skill
	PART – B Long Answer (Attempt any FOUR) Marks: 4 x 12=48		
9	a)Explain in detail about Poisson processes. b).Let X and Y be random variables with joint density function $f(x,y) = f(x) = \begin{cases} 4xy, & 0 < x,y < 1 \\ 0, & otherwise \end{cases}$ Find expected value of $Z = \sqrt{X^2 + Y^2}$.	Explain and Obtain	Remember, Understand, Analyze, Apply, skill
10	a)Given that $X(t)$ is a random process with mean $\mu(t) = 3$ and autocorrelation function $R(t_1, t_2) = 9 + 4e^{-0.2\|t_1 - t_2\|}$. Determine the mean, variance and covariance of the random variables $Y = X(5)$ and $Z = X(8)$. b) Explain in details about birth and death process.	Explain, Problem, Obtain	Remember, Understand, Analyze, Apply, skill
11	a)If a random variable X has the moment generating function $M_X(t) = \frac{2}{2-t}$, determine the variance of X. b)Derive the Generating functions relations for Mean and variance	Problem, Explain	Remember, Understand, Analyze, Apply, skill
12	a)A random process $X(t)$ is defined by $X(t) = 2\cos(2\pi t + Y)$, where Y is a discrete random variable with $P(Y = 0) = \frac{1}{2}, \left(Y = \frac{\pi}{2}\right) = \frac{1}{2}$, Find $\mu_x(1)$ and $R_{xx}(0,1)$ b) Consider the random process $X(t) = \cos(t + \emptyset)$ where $\emptyset$ is a random variable with density function $(\emptyset) = \frac{1}{\pi}, -\frac{\pi}{2} < \emptyset < \frac{\pi}{2}$. Check whether the process is stationary or not Stationary.	Problem, Explain, obtain	Understand, Analyze, Apply, skill

13	a)Explain in detail about residual life time processes. b) A college student X has the following study habits. If he studies one night, he is 70% sure not to study the next night. If he does not study one night, he is only 60% sure not to study the next night also. Find (i) the transition probability matrix. (ii) how often he studies in the long run.	Explain, Problem	Analyze, Apply, skill
	Write short notes on any **THREE** of the following: (3 x 4 =12) a) Irreducible Markov Chain b) Ergodic c) Steady State Distribution d) Brownian motion **e)** Basic limit theorem of Markov Chain	Note	Remember, Understand, Analyze, Apply, skill

ELECTIVES					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM06E	Advanced Topics in Differential Equations	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Illustrate some applications of linear algebra concepts to linear homogenous and non-homogenous differential equations.	Remember
CO 2	Understand the definitions of ordinary and singular (regular and irregular) points to study the method of Frobenius for the solution of second-order differential equations	Understand
CO 3	Apply the knowledge of Laplace's equation and its properties to solve the different boundary value problems of ordinary differential equations.	Apply
CO 4	Illustrate the application of partial differential equations to study the behavior of the motion of vibrating stringsby using the Sturm-Liouville problem.	Analyze
CO 5	Explain the method of separation of variables to solve one-dimensional heat, wave, and Laplace equations.	Skill

b. Syllabus

Units	**Content**	**Hrs.**
I	System of first order differential equations: Algebraic properties of solutions of Linear equations, Application of Linear algebra to Differential Equations, Eigenvalue and Eigen vector method, solutions of linear non homogeneous Equations. Power series solution of second order linear differential equations; regular singular points, the method of Frobenius**.**	20
II	Laplace Equation: Boundary value problems, Maximum and Minimum Principles for Laplace Equation, uniqueness and continuity theorems, Dirichlet problem for a circle, Neumann problem for a circle, Theory of Green's function for Laplace's equation**.**	15
III	One Dimensional Wave equation: vibrations of a finite string, vibrations of an infinite string, vibration of a semi-infinite string, D'Alembert's solution, Separable method, existence and uniqueness of solution, Riemann method. Duhamel's principle for wave equations	15
IV	Heat Conduction Problem in a finite rod, Heat Conduction problem for an infinite rod, Separable method, Existence and Uniqueness of the solution. Duhamel's principle for heat equations. Hadamard's definition of wellposedness**.**	10
	References: **1.** 1 M. Braun, Differential Equations and their Applications, Fourth Edition, Springer, 1993. **2.** E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India Ltd., 2002.	

	3. F. John, Partial Differential Equations, second edition, Springer-Verlag, 1978. **4.** L.C. **Evans, Partial Differential Equations, AMS, 2010.**	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	2	1	1	0
CO2	3	1	1	0	1
CO3	3	1	1	0	0
CO4	3	2	2	1	1
CO5	3	2	1	0	1

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A (Objective - 10 x 1 = 10 marks)	2	2	2	2	2
Part – B (Short Answer - 5 x 4 = 20 marks)	10	10	-	-	-
Part – C (Essay- 3 x 10 = 30 marks)	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

ELECTIVES					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM07E	Mathematical Modeling in Biology	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Understand basic knowledge about biological systems in order to model them and also use techniques and tools of applied mathematics to describe and solve various biological problems.	Remember
CO 2	Examine the modeling foundations (defining goals, identifying variables, parameters, and assumptions) of simple discrete and continuous models of biological phenomena.	Understand
CO 3	Formulate the mathematical models based on exponential and logistic growth models for the cell cycle, cell division, and tumor cell.	Apply
CO 4	Apply the concepts of linear stability and phase plane analysis to predict the behavior of living systems.	Analyze
CO 5	Demonstrate an understanding of powerful mathematical tools such as ODE and PDE and apply them to infectious disease modeling (SI, SIR, SEIR, HIV), models for animal dispersal, pattern formation in biological systems, cellular chemotaxis model mechanisms, and predator-prey model in population biology.	Skill

b. Syllabus

Units	**Content**	**Hrs.**
I	Introduction, motivation and background biology; Continuous growth models; Insect outbreak model-Spruce Budworm; Delay models; Tumour cell growth; Model with spatial Heterogeneity; Tumour cell spreading dynamics in vitro-parameter estimation.	10
II	Models with age distribution; Logistic-type model -Chaos; Discrete delay models: Fishery management model; Predator-Prey models, Analysis of predator-prey model with limit cycle periodic behavior; Threshold phenomena; Lotka-Volterra systems.	15
III	Competition models; Mutualism or Symbiosis, Waves in biology; Spiral waves; Traveling wave trains in reaction diffusion systems; Spiral wave solutions of $\boldsymbol{\lambda}$–$\boldsymbol{\omega}$ reaction diffusion systems; Enzyme kinetics; Law of mass action; Basic model for the dynamics of nerve membranes; Infectious Diseases; Simple epidemic models (SI, SIR, SEIR, and HIV), Classical Kermack-McKendrick model.	20
IV	Biological oscillators; Singular perturbation analysis for biological applications; Analysis of the phase shift equation and application to the coupled Belousov-Zhabotinskii reactions, Reaction diffusion equations; Models for animal dispersal; Pattern formation in biological systems; Cell-chemotaxis model	15
	References: **1.** J. D. Murray, Mathematical Biology I: An Introduction	

	(Interdisciplinary Applied Mathematics), Third Edition, Springer-Verlag Berlin Heidelberg, New York, 2002. **2.** J. D. Murray, Mathematical Biology II: Spatial Models and Biomedical Applications, (Interdisciplinary Applied Mathematics), Third Edition, Springer-Verlag Berlin Heidelberg, New York, 2003. **3.** L. Edelstein-Keshet, Mathematical Models in Biology (Classics in Applied Mathematics), Society for Industrial and Applied Mathematics (SIAM), Philadelphia, New York, 2005.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	2	1	0	0
CO2	3	2	3	0	1
CO3	3	2	2	1	1
CO4	3	2	1	1	0
CO5	3	3	3	2	0

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 10 x 1 = 10 marks)**	2	2	2	2	2
Part – B **(Short Answer - 5 x 4 = 20 marks)**	10	10	-	-	-
Part – C **(Essay- 3 x 10 = 30 marks)**	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

SAM07E - Mathematical Modelling in Biology

Sl. No.	Model Questions	Specification	Level
	Part – A: Objective Type **Multiple Choice** **Marks: 10 x 1 = 10**		
1	In which phase nuclear DNA replicates? (A) G1 phase (B) S phase (C) G2 phase (D) M phase	Recall	Remember
2	As of 1995, the human population was expected to double within 25 years. Calculate the intrinsic growth rate 'k' (rounded to 5 decimal places) for the human population. (A) 0.02773 (B) 36.06738 (C) 0.02772 (D) 36.06737	Find	Apply, Skill
3	Which of the following modelled equation represents delayed logistic population dynamics? (A) $\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right)$ (B) $\frac{dN}{dt} = r_B N\left(1-\frac{N}{K_B}\right) - \frac{BN^2}{(A^2+N^2)}$ (C) $\frac{dN}{dt} = rN$ (D) $\frac{dN}{dt} = rN(t)\left[1-\frac{N(t-T)}{K}\right]$	Examine	Remember, Analyze
4	What is the characteristic shape of a curve illustrating logistic growth model? (A) J-shaped (B) U-shaped (C) S-shaped (D) L-shaped	Recall, Examine	Remember, Understand
5	The von Foerster equation for population models with age structure is given by $\frac{\partial n(a,t)}{\partial t} + \frac{\partial n(a,t)}{\partial a} = -\mu(a,t)n(a,t)$, a, $t \geq 0$ and its boundary conditions are $n(0,t) = \int_0^\infty b(a,t)\, n(a,t)dt$, $t \geq 0$ and $n(a,0) = f(a)$, $a \geq 0$, where b and μ denotes the birth and death rates which are functions of age 'a'. What is the solution of this modelled equation when $a > t$ (A) $n(t,a) = f(a-t)\; exp\left(\int_{a-t}^{a} \mu(\sigma)d\sigma\right)$ (B) $n(t,a) = n(t-a,0)\; exp\left(-\int_0^a \mu(\sigma)d\sigma\right)$ (C) $n(t,a) = f(a-t)\; exp\left(-\int_{a-t}^{a} \mu(\sigma)d\sigma\right)$ (D) $n(t,a) = n(t-a,0)\; exp\left(\int_0^a \mu(\sigma)d\sigma\right)$	Solve, Prove	Analyze, Understand, Skill
6	In modelled equation, $\frac{\partial G_1}{\partial t}(x,t) = 2^2 bM(2x,t) - \left(k_1 + \mu_{G_1}\right)G_1(x,t)$, where the time variable 't' is measured in hours while x, relative DNA content, describing G1 phase, the first term on R.H.S is the source term provided by (A) the influx of newly divided daughter cells from G1 phase (B) the loss of cells from G1 phase (C) the influx of newly divided daughter cells from M phase (D) the loss of cells from G2 phase	Find, Recall	Analyze, Understand, Skill
7	Consider the Lotka-Volterra predator-prey model, $\frac{dN}{dt} = N(a-bP)$ and $\frac{dP}{dt} = P(cN-d)$, where N(t) and P(t) are prey and predator populations, respectively. All possible equilibrium points for the above	Check, Find	Analyze, Apply, Skill

	system are____. (A) (0, 0), $(-\frac{d}{c}, \frac{a}{b})$, (B) (0, 0), $(\frac{d}{c}, -\frac{a}{b})$, (C) (0, 0), $(\frac{d}{c}, \frac{a}{b})$, (D) (0, 0), $(-\frac{d}{c}, -\frac{a}{b})$		
8	Which of the following is not an assumption of the Lotka-Volterra predator-prey dynamics model? (A) the prey population increases exponentially when the predator is absent (B) the predator population increases exponentially when the prey is absent (C) the predator population will starve in the absence of prey population (D) the predators can consume infinite quantities of prey	Recall, Verify	Aanlyze, Understand
9	Assuming that the total human population of the earth grows exponentially. How much time (in years) needed for this population to increase by a factor of 2 per year? (Rounded to 1 decimal place) (A) 34 (B) 35 (C) 15 (D) 16	Find	Apply, skill
10	The main function of insulin is to (A) allow the absorption of nutrients through the small intestine (B) enable glucose to enter the body cells (C) break down protein (D) speed up the concentrations of the stomach	Recall	Remember, understand
	PART – B Short Answer **The answer should not exceed 200 words Marks:5 x 4 = 20**		
11	**a)** Write any five social factors that affect the population growth. **(or)** **b)** If $n(t,a) = e^{\gamma t} r(a)$ be the solution of the age distribution model equation, $\frac{\partial n}{\partial t} + \frac{\partial n}{\partial a} = -\mu(a)n$, satisfies the age boundary condition, $n(t,0) = \int_0^\infty b(a)n(t,a)da$, then prove that $\int_0^\infty b(a)\, exp\left[-\gamma a - \int_0^a \mu(s)ds\right]da = 1$.	Recall, Prove,	Remember, Understand, Analyze
12	**a)** A model of a fishery with harvesting is $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - \frac{hN}{A+N}$ where $N(t)$ is the population of fish at time t and r, K, h, A are positive parameters. Explain why this is a reasonable model and give a biological interpretation of each of the parameters. **(or)** **b)** Using the dimensionless quantities, $\tau = rt$ and $U = \frac{N}{K}$, show that the differential equation (as given in the above question, part(a)) can be put indimensionless form: $\frac{dU}{d\tau} = U(1-U) - \frac{HU}{\alpha+U}$,where $\alpha = \frac{A}{K}$,$H = \frac{h}{rK}$.	Explain, Examine, Derive	Understand, Analyze, Apply, Skill
13	**a)** Write any five differences between an infectious and non-infectious disease. **(or)** **b)** Explain the SIR epidemic model for infectious disease.	Recall, Explain,	Understand, Analyze, Remember
14	a) The Lotka-Volterra model for species competition is given by the equations, $\frac{dN_1}{dt} = r_1 N_1\left[1 - \frac{N_1}{K_1} - \frac{\beta_{12}N_2}{K_1}\right]$ and $\frac{dN_2}{dt} = r_2 N_2\left[1 - \frac{N_2}{K_2} - \frac{\beta_{21}N_1}{K_2}\right]$, where N_1 and N_2 arethe population densities of species 1 and 2.	Examine, Recall, Explain	Understand, Analyze, Apply

	i) What are the meanings of the parameters $r_1, r_2, K_1, K_2, \beta_{12}$, and β_{21}? ii) Suppose only species 1 is present. What has been assumed about itsgrowth? iii) What kind of assumption has been made about the effect of competition on thegrowth rate of each species? **(or)** **(b)** Derive diffusion equation by extending the simple random walk.		
	PART – C Essay Answer **The answer should not exceed 400 words Marks: 3 x 10 = 30**		
15	**a)** i) Distinguish between exponential and logistic population growth. Give the equations for each. **ii)** Solve the logistic equation for population growth. **(or)** **b)** **i)**Define the delayed logistic equation of population dynamics. **ii)** Describe the mathematical model for tumor cell growth.	Define, Recall, Examine	Remember, Understand, Analyze
16	**a)** Perform nondimensionalization and a phase plane analysis of model considered inquestion 5(a) above by introducing the transformations $u(\tau) = \frac{cN(t)}{d}$, $v(t) = \frac{bP(t)}{a}$,$T = at, \alpha = \frac{d}{a}$. **(or)** **(b)** Consider the SIS model including vital dynamics, $\frac{dS}{dt} = -\lambda IS + \gamma I + \mu - \mu S$ and $\frac{dI}{dt} = \lambda IS - \gamma I - \mu I$,where S and I represent susceptible and infectiveclasses respectively and $S(0) = S_0 > 0$, $I(0) = I_0 > 0$, $S(t) + I(t) = 1$. Then show that thesolution (i.e., infection with time t) of given system of differential equations is $$I(t) = \begin{cases} \frac{e^{(\gamma+\mu)(\sigma-1)t}}{\frac{\sigma[e^{(\gamma+\mu)(\sigma-1)t}-1]}{(\sigma-1)} + \frac{1}{I_0}}, for\ \sigma \neq 1 \\ \frac{1}{\lambda t + \frac{1}{I_0}}, \quad \text{for} \sigma = 1 \end{cases}$$ where σ is defined as the contact number, $\sigma = \frac{\lambda}{\gamma+\mu}$.	, Prove, Problem, Explain	Understand, Analyze, Apply, Skill
17	**a)** Consider the partial differential equation $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$, where $c(x,t)$ is the concentration of the species and D is its diffusivity, which is a constant. If we release an amount Q of particle per unit area at $x = 0$ at $t = 0$, i.e., $c(x,0) = Q\delta(x)$, where $\delta(x)$ is the Dirac delta function, then show that solution of the given equation is $c(x,t) = \frac{Q}{2(\pi Dt)^{1/2}} e^{-\frac{x^2}{4Dt}}$, $t > 0$. **(or)** **(b)** Describe the mathematical model of chemotactic process.	Problem, Prove, Describe	Remember, Understand, Analyze, Apply, skill

ELECTIVES					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM08E	Integral Transforms	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Explain the definition and existence of Laplace transform, Fourier transform, Z, Hankel and Mellin transforms.	Remember
CO 2	Understand the basic properties of Laplace transform, Fourier transform, Z, Hankel and Mellin transforms.	Understand
CO 3	Apply the appropriate transform to solve initial value problems, boundary value problems, signal processing	Apply
CO 4	Analyze the suitable transform to solve ODEs, PDEs and Difference equations.	Analyze
CO 5	Get deep knowledge on solving real time mathematical models by suitable integral transforms.	Skill

b. Syllabus

Units	**Content**	**Hrs.**
I	The Fourier Transform: Algebraic Properties, Convolution, Translation, Modulation. Analytical Properties of Fourier transforms, Transform of Derivatives and Derivatives of Transform, Perseval Formula, Inversion theorem, Plancherel's theorem, Application to solving Ordinary and Partial Differential Equations**.**	15
II	The Laplace transform: Algebraic Properties of Laplace Transform, Transform of Derivatives and Derivatives of Transform. The Inversion Theorem, Evaluation of inverse transforms by residue. Asymptotic expansion of inverse transform, Application to solving PDE and Integral Equation	15
III	The Hankel transform: Elementary properties, Inversion theorem, Transform of derivatives of functions, Perseval relation, Relation between Fourier and Hankel transform, The Mellin transform: Properties and Evaluation of Transforms, Convolution Theorem for Mellin Transforms. Solving PDE by Hankel transform and solving integral equations by Mellin Transform	20
IV	The Z-Transform: Properties of the region convergence of the Z-transform. Inverse Z-Transform for Discrete-Time Systems and Signals, Signal Processing and Linear System	10
	References: **1.** D.Loknath, Integral Transforms and their Application, CRC Press, 1995**.** **2.** R. S. Pathak, Integral Transform for Generalized Functions and their Applications. Gordon and Breach Science Publishers, 1997**.** **3.** F.C**.** Titchmash, Introduction to the theory of Fourier Integrals,	

	Oxford Press, 1937. **4.** E.J. Watson, Laplace Transforms and Application, Van Nostland Reinhold Co. Ltd., 1981. **5.** E.I. Jury, Theory and Application of Z-Transform, John Wiley and Sons, 1973. **6.** W. Rudin, Real and Complex Analysis, Mc. Graw Hill Inc., 1987.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	3	2	1	3
CO2	3	1	2	1	3
CO3	3	2	2	1	3
CO4	3	2	2	1	3
CO5	3	3	3	3	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 10 x 1 = 10 marks)**	2	2	2	2	2
Part – B **(Short Answer - 5 x 4 = 20 marks)**	10	10	-	-	-
Part – C **(Essay- 3 x 10 = 30 marks)**	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

	Part – A: Objective Type **Multiple choice 10 x 1 = 10**		
1	Find the Sufficient condition for f(x) to have Fourier transform A. absolutely integrable B. integrable C. square integrable. D.None of the above	Identify	Remember
2	Let $F(k) = \mathcal{F}(f(x))(k)$. Then evaluate $\mathcal{F}(f(ax))(k)$. A.$\frac{1}{a}F(\frac{k}{a})$B. $aF(\frac{k}{a})$ C. $\frac{1}{a}F(ka)$ D. aF(ka).	Recall	Remember
3	Choose the incorrect option A. Laplace transform of t^{-n} does not exist. B. Laplace transform of e^{t^2} does not exist. C. $\|t^n \sin at\|$ is not exponentially bounded. D. Choice A and B are correct.	Recall	Remember
4	Find the Sufficient condition for f(x) to have Laplace transform A. Exponentially bounded C. Piecewise Continuous B. A and C D. None of the above	Evaluate	Remember
5	Evaluate Z{(1,1,1,1,1,…)} A. $\frac{1}{z-1}$ B. $\frac{1}{z+1}$ C. $\frac{z}{z-1}$ D. $\frac{z}{z+1}$	Recognize	Remember
6	Evaluate Z transform of $H(n) - H(n-2)$ A. $\left(1+\frac{1}{z}\right)$B. $\left(1-\frac{1}{z}\right)$ C. $\left(-1+\frac{1}{z}\right)$D. $-\left(1+\frac{1}{z}\right)$	Evaluate	Remember
7	Evaluate $\mathcal{H}_0(1)$ A. $\delta(\lambda)/\lambda$ B $\delta(\lambda)$ C. $\lambda\delta(\lambda)$ D. $\delta(\lambda)/\lambda^2$	Recall	Remember
8	Find the first order Hankel transform of e^{-ar}. A. $\frac{\lambda}{\sqrt{(\lambda^2+a^2)}^{\,3}}$B.$\frac{\lambda}{\sqrt{(\lambda^2+a^2)}^{\,5}}$ C. $\frac{\lambda}{\sqrt{(\lambda^2-a^2)}^{\,3}}$ D. $\frac{\lambda}{\sqrt{(\lambda^2-a^2)}^{\,5}}$	Recall	Remember
9	Evaluate the Mellin transform of $f(t) = e^{-pt}$. A. $p^{-s}\Gamma(s)$ B. $\Gamma(s)$ C. $p^{s}\Gamma(s)$ D. $p^{-2s}\Gamma(s)$	Evaluate	Remember
10	By using Mellin transform, find the sum of the infinite series $$\sum_{n=1}^{\infty} \delta(t-an), a > 0$$ $A.$ $a^{s-1}\mathcal{G}(1-s)$ B . $a^{s+1}\mathcal{G}(1-s)$ C. $a^{s+1}\mathcal{G}(1+s)$ D.$a^{s-1}\mathcal{G}(1+s)$	Evaluate	Understand
	PART – B Short Answer **The answer should not exceed 200 words 5 x 4 = 20**		
11	a) Evaluate $\mathcal{L}^{-1}\{\frac{1}{\sqrt{s}(s-a)}\}(t)$ (or) b)Find the Fourier transform of $f(x) = e^{-ax^2}$. Further, show the self reciprocal of $f(x)$ for suitable a value.	Evaluate	Understand
12	a) Find the Laplace transform for rectified sine wave function. (or)	Evaluate	Apply

	b) Find the solution for the initial value problem by Laplace transform. $y'(t) + 5y(t) = \sin t , t \epsilon (0,10).$ $y(0) = 3.$		
13	a) Find the n[th]-term of the Fibanacci series by suitable transform method. (or) b) Verify the convolution theorem of Z- transform for u_n =n and v_n =a, $a > 0$.	Examples	Apply
14	a) Show the following by Mellin Transform $\sum_{n=1}^{\infty}\left(a_n \cos\frac{kn}{n^2}\right) = \frac{k^2}{4} - \frac{k\pi}{2} + \frac{\pi^2}{6}$ (or) b) Show that $\mathcal{H}_n\{e^{-ar} f(r)\} = \mathcal{L}\{r f(r) J_n(kr)\}(s)\|_{s=a}$ Where $\mathcal{L}$ stands for the Laplace transformation. Further deduce the result for $n = 0 and\ f(r) = 1$.	Discuss	Apply
	PART – C Essay Answer **The answer should not exceed 400 words 3 x 10 = 30**		
25	a) Solve the second order difference equation by Z - transform. $3f(n + 2) - 2f(n + 1) - f(n) = 0$ $f(0) = 1; \quad f(1) = 2.$ Also show that $f(n) \rightarrow 7/4$ as $n \to \infty$. (or) b) Find the solution of the axis symmetric diffusion problem with initial temperature f(r) and diffusivity constant k by Hankel transform.	Describe	Analyse
26	a) For $x \in \mathbb{R}, t > 0$, let $u(x,t)$ denotes the number of neutrons per unit volume and unit time, which reach the age t. Consider the problem of slowing down neutrons in an infinite medium with a source of neutrons $\delta(x)\delta(t)$. Find u(x,t) by means of Fourier transformation with initial data $\delta(x)$ and the boundary condition that the solution approaches zero as $\|x\| \to \infty$. (or) b) Use Z- transform to show the following: i.) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, -\infty < x < \infty$ ii.) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{n+1} = \log(1 + x)$.	Evaluate	Apply
27	a) Discuss the harmonic oscillator equation and solve by means of Laplace transformation. (or) b) Find the solution for the vibration of a semi infinite string with initial displacement f(x) and initial velocity g(x) by suitable transformation.	Demonstrate	Skill

ELECTIVES					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM09E	Machine Learning	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	Level
CO 1	Have a good understanding of the fundamental issues and challenges of machine learning: data, model selection, model complexity, etc.	**Remember**
CO 2	Have an understanding of the strengths and weaknesses of many popular machine learning approaches. Understand the mathematical fundamentals that are prerequisites for a variety of courses like Data Mining, Network protocols, analysis of Web traffic, Computer security, Bioinformatics and Machine Learning.	**Understand**
CO 3	Appreciate the underlying mathematical relationships within and across Machine Learning algorithms and the paradigms of supervised and un-supervised learning.	**Apply**
CO 4	Be able to design and implement various machine learning algorithms in a range of real-world applications.	**Analyze**
CO 5	Get exposure to a wide variety of mathematical concepts used in computer science discipline like probability, Graph Theory for solving problems, sampling and estimation.	**Skill**

b. Syllabus

Units	Content	Hrs.
I	Introduction to Machine Learning, Different Forms of Learning, Linear Regression, Ridge Regression, Lasso, Bayesian Regression, Regression with Basis Functions	15
II	Instance-Based Classification**,** Linear Discriminant Analysis, Logistic Regression, Large Margin Classification, Kernel Methods, Support Vector Machines, Multi-class Classification, Classification and Regression Trees	15
III	Neural Networks**:** Multi-layer Networks, Back-propagation, Multi-class Discrimination, Training Procedures, Localized Network Structure, Deep Learning. Graphical Models: Hidden Markov Models, Bayesian Networks, Markov Random Fields, Conditional Random Fields	15
IV	Ensemble Methods: Boosting - Adaboost, Gradient Boosting, Bagging - Simple Methods, Random Forest, Clustering: K-Medoids, CLARA, DENCLUE, DBCSAN**.**	15
	References: **1.** C. Bishop, Pattern Recognition and Machine Learning, Springer - Verlag, 2006**.**	

	2. T. Mitchell, Machine Learning, McGraw Hill Education, 2017. 3. R.O. Duda, P.E. Hart and D.G. Stork, Pattern Classification, Wiley-Black-well; 2nd edition, 2000. 4. T. Hastie, R.Tibshirani and J. Friedman, Elements of Statistical Learning, 2nd Edition, Springer, 2009. 5. Han, Jiawei, Jian Pei, and Micheline Kamber. Data mining: concepts and techniques. Elsevier, 2011.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	3	3	3	2
CO2	3	3	3	3	3
CO3	3	3	3	3	3
CO4	2	2	1	3	2
CO5	2	2	1	3	2

d. Evaluation Scheme

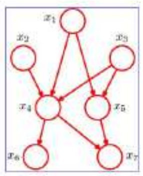
	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A (Objective - 6 x 2 = 12 marks)	4	4	2	2	2
Part – B (Short Answer - 4 x 12 = 48 marks)	8	8	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

<table>
<tr><th></th><th>Model Questions</th><th>Specification</th><th>Level</th></tr>
<tr><td></td><td>Part – A: Short Answer (Attempt any SIX)
Marks: 6 x 2 = 12</td><td></td><td></td></tr>
<tr><td>1</td><td>List the different types of statistical approaches in Machine Learning?</td><td>State</td><td>Analyze, Apply</td></tr>
<tr><td>2</td><td>Find the accuracy and the error, if the samples manufactured are 600 in which 540 are good items.</td><td>State</td><td>Remember,</td></tr>
<tr><td>3</td><td>Represent the following directed graphical model in terms of Probabilistic Graphical Model
</td><td>Problem, Obtain</td><td>Analyze, Apply,</td></tr>
<tr><td>4</td><td>Draw a decision tree for Instance based learning technique.</td><td>Recall, State</td><td>Remember, Understand</td></tr>
<tr><td>5</td><td>Define margin in support vector machines.</td><td>Recall, State</td><td>Remember</td></tr>
<tr><td>6</td><td>What is the use of ROC curve in classification?</td><td>Recall</td><td>Analyze, Apply</td></tr>
<tr><td>7</td><td>What do you mean by Supervised and Unsupervised Algorithms</td><td>Define</td><td>Remember</td></tr>
<tr><td>8</td><td>Define sensitivity of a classifier.</td><td>Recall, State</td><td>Apply, skill</td></tr>
<tr><td></td><td>PART – B Long Answer (Attempt any FOUR)
Marks: 4 x 12=48</td><td></td><td></td></tr>
<tr><td>9</td><td>a) Explain about Instance Based Learning Classification technique.

b) Describe Artificial Neural Network and discuss back propagation method for training the network.</td><td>Problem, Explain</td><td>Remember, Understand, Analyze, Apply, skill</td></tr>
<tr><td>10</td><td>a)Explain the concept of Bagging and Boosting.

b)Find the log-odds ratio and test the significance at 5% level of significance for the following data(given chi square value is 3.8401)
<table><tr><th></th><th>Muted</th><th>Not Muted</th></tr><tr><td>Affected</td><td>23</td><td>117</td></tr></table></td><td>Explain, Problem, Obtain</td><td>Remember, Understand, Analyze, Apply, skill</td></tr>
</table>

<table>
<tr><td></td><td><table><tr><td>Non Affected</td><td>6</td><td>210</td></tr></table></td><td></td><td></td></tr>
<tr><td>11</td><td>a)Find the following measures of multi classification i) Specificity ii) Precision iii) Recall
<table>
<tr><th></th><th>Parrot</th><th>Swan</th></tr>
<tr><td>Parrot</td><td>25</td><td>13</td></tr>
<tr><td>Swan</td><td>14</td><td>31</td></tr>
<tr><td>Pigeon</td><td>6</td><td>9</td></tr>
</table>
b)Describe K-nearest classifier. State its advantages and disadvantages.</td><td>Problem, Explain</td><td>Remember, Understand, Analyze, Apply, skill</td></tr>
<tr><td>12</td><td>a)Using Logistic Regression calculate the probability of the given dataset who has worked in IT industry for 40 hours.
<table>
<tr><th>Number of hours worked</th><th>Promotion (No-0 and Yes-1)</th></tr>
<tr><td>25</td><td>0</td></tr>
<tr><td>18</td><td>0</td></tr>
<tr><td>40</td><td>1</td></tr>
<tr><td>35</td><td>1</td></tr>
<tr><td>36</td><td>1</td></tr>
</table>
b) Explain support vector classification.</td><td>Problem, Explain, obtain</td><td>Understand, Analyze, Apply, skill</td></tr>
<tr><td>13</td><td>a)Describe LASSO. State its advantages.
b)Using Naïve Bayes classifier find the following : P(No); P(Yes); P(Status=Recovered |No); P(Vaccinated=Yes |Yes); P(Vaccinated = No |Yes); P(Status=Died |No) ; P(Status=Recovered |Yes)
<table>
<tr><th>ID</th><th>Vaccinated</th><th>Status</th><th>Amount Paid</th><th>Evaluation</th></tr>
<tr><td>1</td><td>Yes</td><td>Suspected</td><td>100K</td><td>No</td></tr>
<tr><td>2</td><td>No</td><td>Recovered</td><td>150K</td><td>No</td></tr>
<tr><td>3</td><td>No</td><td>Suspected</td><td>80K</td><td>No</td></tr>
<tr><td>4</td><td>Yes</td><td>Recovered</td><td>90K</td><td>No</td></tr>
<tr><td>5</td><td>No</td><td>Died</td><td>60K</td><td>Yes</td></tr>
<tr><td>6</td><td>No</td><td>Recovered</td><td>200K</td><td>No</td></tr>
<tr><td>7</td><td>Yes</td><td>Died</td><td>110K</td><td>No</td></tr>
<tr><td>8</td><td>No</td><td>Suspected</td><td>70K</td><td>Yes</td></tr>
<tr><td>9</td><td>No</td><td>Recovered</td><td>89K</td><td>No</td></tr>
<tr><td>10</td><td>No</td><td>Suspected</td><td>95K</td><td>Yes</td></tr>
</table></td><td>Explain, Problem</td><td>Analyze, Apply, skill</td></tr>
</table>

14	Write short notes on any **THREE** of the following: (3 x 4 =12) a.Lasso Regression b.Classification and Regression Trees. c.Bayesian Regression d..Ridge Regression e.DENCLUE fLogistic Regression as a classifier	Note	Remember, Understand, Analyze, Apply, skill

ELECTIVES					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM16E	Computational Introduction to Number Theory	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Express the concept of numbers, congruence and all the basics facts on number theory	Remember
CO 2	Understand the techniques to compute with large integers, distribution of primes, finite and discrete probability distributions.	Understand
CO 3	Apply mathematical ideas and concepts within the context of computational number theory.	Apply
CO 4	Analyze the number theoretic techniques.	Analyze
CO 5	Solve computational problems in number theory.	Skill

b. Syllabus

Units	**Content**	**Hrs.**
I	Basic properties of the integers-Divisibility and primality, Ideals and greatest common divisors, Some consequences of unique factorization; Congruences - Definit-tions and basic properties, Solving linear congruences, Residue classes, Euler's phi function, Fermat's little theorem, Arithmetic functions and Mobius inversion	15
II	Computing with large integers -Asymptotic notation, Machine models and complexity theory, Basic integer arithmetic, Computing in Z_n, Faster integer arithmetic. Euclid's algorithm- The basic and extended Euclidean algorithms, computing modular inverses and Chinese remaindering, Speeding up algorithms via modular computation, Rational reconstruction and applications	15
III	The distribution of primes - Chebyshev's theorem on the density of primes, Bertrand's postulate, Mertens' theorem, The sieve of Eratosthenes, The prime number theorem. Quadratic residues and quadratic reciprocity; Quadratic residues, The Legendre symbol, The Jacobi symbol. Computational problems related to quadratic residues; Computing the Jacobi symbol, testing quadratic residuosity, Computing modular square roots, The quadratic residuosity assumption	15
IV	Finite and discrete probability distributions -Finite probability distribute-ons**:** basic definitions, Conditional probability and independence, Random variables, Expectation and variance, Some useful bounds, The birthday paradox, Hash functions, Statistical distance, Discrete probability distributions	15
	References: **1.** Victor Shoup, A computational introduction to number theory and algebra Cambridge University Press, Cambridge, 2005**.**	

	2. T. M. Apostol, Introduction to Analytic Number Theory, Springer-Verlag, New York, 1976. 3. D.Bressoud and S. Wagon, A course in Computational Number Theory, Key College Publishing, Emeryville, CA; in cooperation with Springer-Verlag, New York, 2000.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	01	2	1	2
CO2	3	2	2	1	2
CO3	3	2	2	3	3
CO4	3	2	2	1	2
CO5	3	3	3	3	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 10 x 1 = 10 marks)**	2	2	2	2	2
Part – B **(Short Answer - 5 x 4 = 20 marks)**	10	10	-	-	-
Part – C **(Essay- 3 x 10 = 30 marks)**	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

ELECTIVES					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM17E	Regression Analysis	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Recall the basics of linear regression model and underlying assumptions.	**Remember**
CO 2	Explain parameter estimation, testing of hypothesis and confidence intervals in various regression models.	**Understand**
CO 3	Perform parameter estimation and testing of hypothesis; and construct confidence intervals. Implement variable selection methods to identify appropriate model for further analysis.	**Apply**
CO 4	Detect problems like multicollinearity and outliers in data. Estimate regression parameters in the presence of multicollinearity and outliers.	**Analyze**
CO 5	Interpret the results of Regression analysis.	**Skill**

b. Syllabus

Units	**Content**	**Hrs.**
I	Multiple regression model, least squares estimate (LSE), Properties of LSE, Hypothesis testing, confidence and prediction intervals, General linear hypothesis testing. Dummy variables and their use in regression analysis. Residuals and their properties, residual diagnostics. Transformation of Variables**:** VST and Box-Cox power transformation	15
II	Variable Selection Procedures**:** R-square, adjusted R-square, Mallows' Cp, forward, backward and stepwise selection methods, AIC, BIC. Multicollinearity**:** Consequences, detection and remedies, ridge regression. Autocorrelation: sources, consequences, detection (Durbin-Watson test) and remedies. Parameter estimation using Cochrane-Orcutt method**.**	15
III	Nonlinear regression models**:**Nonlinear least squares, Transformation to a linear model, Parameter estimation in a nonlinear system, Statistical inference in nonlinear regression. Polynomial regression model, piecewise polynomial fitting, nonparametric regression: kernel and locally weighted regression	15
IV	Robust Regression**:** Influential observations, leverage, outliers, methods of detection of outliers and influential observations, estimation in the presence of outliers: M-estimator, Huber loss function, breakdown point, influence function, efficiency, Asymptotic distribution of M-estimator (Statement only), Mallows' class of estimators	15
	References: **1.** Draper N.R. and Smith**,** H**.** (1998)**:** Applied Regression Analysis. 3rd ed Wiley **2.** Wiesberg, S**.** (1985)**:** Applied Linear Regression, Wiley**.**	

	3. Kutner, Neter, Nachtsheim and Wasserman (2003): Applied Linear Regression Models, 4th Edition, McGraw-Hill. 4. Montgomery, D.C., Peck, E.A.and Vining, G.(2012): Introduction to Linear Regression Analysis, 5th Ed. Wiley. 5. Cook R.D. andWiesbergS. (1982): Residuals and Influence in Regression. Chapman and Hall. 6. Birkes, D.and Dodge, Y. (1993). Alternative methods of regression, John Wiley and Sons. 7. Huber, P. J. and Ronchetti, E. M.(2011) Robust Statistics, Wiley, 2nd Edition. 8. Seber, G. A., Wild, C.J. (2003). Nonlinear Regression, Wiley.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	2	1	1	1
CO2	2	3	2	1	2
CO3	2	3	2	3	3
CO4	2	3	2	3	3
CO5	1	2	3	1	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Short Answer - 6 x 2 = 12marks)**	2	2	2	2	2
Part – B **(Long Answer - 4 x 12 = 48 marks)**	10	10	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

g. Rubric for Assignments

Sl. No.	Criteria	100%	75%	50%	25%	0%	Relation to COs
1	**Content 50%**	Ideas are detailed, well developed, supported with specific evidence &facts andexamples	Ideasaredetailed, Developed and supported with evidence and facts mostly specific.	Ideas are presented but not particularly developed or supported;	Content is not sound	Not attended	CO1, CO2, CO5
2	**Organiza-tion 50%**	Includes title, introduction,statement ofthemain idea withillustration and conclusion.	Includes title, introduction,statementofmainideaand conclusion.	organizational tools are weak or missing	Noorganization	Not attended	CO1, CO2, CO5

h. Rubric for Seminar

Sl. No.	Criteria	100%	75%	50%	25%	0%	Relation to COs
1	**Knowledge and Understanding 50%**	Exceptional knowledge of facts, terms, and concepts	Detailed knowledge of facts, terms, and concepts	Considerable knowledge of facts, terms, and concepts	Minimal knowledge of facts, terms, and concepts	Not Attended	CO3, CO4

2	**Presentation** **50%**	Well Communicated with logical sequences, examples, and references	Communicated with sequences	Just Communicated	No coherent communication	Not Attended	CO3, CO4

i. Model Question Paper

Sl. No.	Model Questions	Specification	Level
	Part – A: Short Answer (Attempt any SIX) **Marks: 6 x 2 = 12**		
1	Comment on the linearity of the model $y = \beta_0 + \beta_1^2 x + \epsilon$.	Example	Understand
2	Explain normal probability plot of residuals.	Explain	Remember, Understand
3	Define AIC. State its limitations.	Define, State	Remember, Understand
4	Describe Durbin-Watson test. State its application.	Recall, State	Remember, Understand
5	Define Dummy variables.	Recall	Remember
6	If there are 4 categorical variables each having 3 categories, then how many dummy variables are required to be incorporated in the regression model?	Problem	Apply, Skill
7	Define R-square. State its interpretation.	Recall, Interpretation	Remember, Understand
8	Prove that R-square≥Adj. R-square.	Prove	Analyze
	PART – B Long Answer (Attempt any FOUR) Marks: 4 x 12=48		
9	a) Define non linear regression model. Give an example. Describe non linear least squares method for estimating parameters in non linear regression models and comment on the problem of local optimum. b) Define intrinsically linear model. Give an example. Describe linearization method for estimating parameters in non linear regression model. Illustrate the same for the model $y = \theta_1 e^{\theta_2 x} + \epsilon$. Comment on the choice of starting values and convergence criterion.	Describe, Example, Define, Example, Problem	Remember, Understand, Analyze, Apply
10	a) Describe polynomial regression models. State the considerations for fitting a polynomial regression model to the data.	State, Prove, Define, Problem, Derive	Remember, Understand, Analyze

	b) Define multiple linear regression model. State the assumptions on the model. Derive least squares estimator of regression coefficients.		
11	a) Explain scaled residuals. Describe their use in residual diagnostics. b) Define autocorrealation. Explain its causes and consequences.	Explain, Describe, Define	Remember, Understand, Analyze
12	a) Define multicollinearity. Give various measures for diagnosing multicollinearity. b) Explain ridge regression.	Define, Give, Explain	Remember, Understand, Analyze
13	a) What is the problem of variable selection? Derive Mallow's Cp criterion. b) Describe estimation of regression parameters in the presence of autocorrelation.	What, Derive, Describe	Remember, Understand, Analyze
14	Write short notes on any **THREE** of the following: (3 x 4 =12) a) Splines for smoothing. b) Kernel Regression. c) Box-Cox transformation. d) Forward Selection Method. e) Testing general linear hypothesis	Note	Remember, Understand, Analyze, Apply, skill

ELECTIVES					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM18E	Generalized Linear Models	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	Level
CO 1	Recall the basics of generalized linear models (GLM) and underlying assumptions.	**Remember**
CO 2	Explain parameter estimation, testing of hypothesis and confidence intervals in GLM.	**Understand**
CO 3	Implement Logistic regression, Poisson regression and NB-2 models for data analysis.	**Apply**
CO 4	Compare the results of different models.	**Analyze**
CO 5	Interpret the results of GLM.	**Skill**

b. Syllabus

Units	Content	Hrs.
I	Generalized linear models**:** concept of generalized linear model, Link function, ML estimation, Quasi-likelihood estimation, large sample tests about parameters, goodness of fit, analysis of deviance. Residual analysis, types of residuals**:** raw, Pearson, deviance, Anscombe, quantile; residual plots. Variable selection**:** AIC and BIC	15
II	Logistic regression**:** logit, probit and cloglog model for dichotomous data with single and multiple explanatory variables, ML estimation, large sample tests about parameters. Hosmer-Lemeshow test, ROC curve. Multilevel logistic regression, Logistic regression for Nominal response: Baseline Category model and ordinal response: Proportional odds model	15
III	Poisson regression**:** ML and Quasi-likelihood estimation of parameters, testing significance of coefficients, goodness of fit, power family of link functions, over dispersion**:**Types, causes and remedies. Negative Binomial regression**:** NB-2 model**.**	15
IV	Generalized linear mixed models (GLMM)**:** Structure of the model, consequences of having random effects, estimation by maximum likelihood, marginal versus conditional models, estimation by generalized estimating equations and conditional likelihood, tests of hypothesis**:** LRT, asymptotic variance, Wald and score test**.**	15
	References: **1.** Hosmer D.W. and Lemeshow S. (2000)**:** Applied Logistic regression, 2nd ED. Wiley NewYork**.** **2.** Agresti A**.** (1990)**:** Categorical Data Analysis. Wiley, New York**.** **3.** R. Christensen (1997) Log-Linear Models and Logistic Regression, Springer. New York**.** **4.** Hilbe, J. (2011)**:** Negative Binomial regression, Cambridge University, Press, 2nd Edition**.**	

	5. McCulloch, C. E.,& Searle, S. R. (2003). Generalized, linear, and mixed models, Wiley series in probability and statistics, New York.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	2	1	1	1
CO2	2	3	2	1	2
CO3	2	3	2	3	3
CO4	2	3	2	3	3
CO5	1	2	3	1	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A (Short Answer - 6 x 2 = 12 marks)	2	2	2	2	2
Part – B (Long Answer - 4 x 12 = 48 marks)	10	10	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

g. Rubric for Assignments

Sl. No.	Criteria	100%	75%	50%	25%	0%	Relation to COs
1	**Content 50%**	Ideas are detailed, well developed, supported with specific evidence &factsandexamples	Ideasaredetailed, Developed and supported with evidence and facts mostly specific.	Ideas are presented but not particularly developed or supported;	Content is not sound	Not attended	CO1, CO2, CO5
2	**Organiza-tion 50%**	Includes title, introduction,statement ofthemain ideawithillustration and conclusion.	Includes title, introduction,statementofmainideaand conclusion.	organizational tools are weak or missing	Noorganization	Not attended	CO1, CO2, CO5

h. Rubric for Seminar

Sl. No.	Criteria	100%	75%	50%	25%	0%	Relation to COs
1	**Knowledge and Understanding 50%**	Exceptional knowledge of facts, terms, and concepts	Detailed knowledge of facts, terms, and concepts	Considerable knowledge of facts, terms, and concepts	Minimal knowledge of facts, terms, and concepts	Not Attended	CO3, CO4
2	**Presentation 50%**	Well Communicated with logical sequences, examples, and references	Communicated with sequences	Just Communicated	No coherent communication	Not Attended	CO3, CO4

i. Model Question Paper

Sl. No.	Model Questions	Specification	Level
	Part – A: Short Answer (Attempt any SIX) **Marks: 6 x 2 = 12**		
1	What is a canonical link? Give an example.	Recall, Example	Remember, Understand
2	Define quantile residual. Give an example.	Recall, Example	Remember, Understand
3	Give a real-life situation where logistic regression is applicable. Clearly state your response and regressors	Example	Understand
4	Define odds ratio in simple logistic regression model. State its interpretation	Recall, Interpretation	Remember, Understand
5	State a test for testing significance of individual regression coefficients in a GLM.	Recall	Remember
6	Define exponential family of distributions. Express PMF Poisson distribution in the general form of exponential family of distributions.	Recall, Express	Remember, Analyze
7	Show that Binomial distribution is a member of exponential family.	Example	Understand
8	What is ROC curve? State its use in logistic regression	What, use	Understand, Analyze
	PART – B Long Answer (Attempt any FOUR) Marks: 4 x 12=48		
9	a) Describe formal structure of GLM. Explain estimation of parameters in a GLM using Maximum likelihood estimation. b) Explain Logistic regression model. Derive Score equation for estimating parameters in simple logistic regression model. State the interpretations of parameters involved in the model.	Describe, Explain, Derive, State	Remember, Understand, Analyze, Apply
10	a) Define deviance. Derive a general expression of deviance for a response distribution belonging to exponential family. Derive Deviance when distribution of response is Normal. b) Describe role of Deviance in testing of hypothesis in GLM.	Derive, Define, Describe	Remember, Understand, Analyze, Apply
11	a) Describe Poisson regression model. Derive Score equations for estimating parameters in a Poisson regression model. b) Explain different types of residuals in GLM. What is the need of residual diagnostics in GLM?	Derive, Explain, Describe	Remember, Understand, Analyze
12	a) Define overdispersion. Describe its sources. b) Why overdispersion is a problem? Justify.	Define, Describe, Why, Justify	Remember, Understand, Analyze
13	a) Define fixed and random effects. Give an example each. What is a	Define,	Remember,

	mixed effect model? Define generalized linear mixed effect model. b) Explain estimation of parameters in a generalized linear mixed model.	What, Example, Explain	Understand, Analyze
14	Write short notes on any **THREE** of the following: (3 x 4 =12) a) Negative Binomial regression model b) Polytomous logistic regression model c) Hosmer-Lemeshow test d) Computational Algorithm for MLE in GLM e) Applications of Poisson Regression	Note	Remember, Understand, Analyze, Apply

ELECTIVES					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM19E	Introduction to Fractional Calculus	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	**Level**
CO 1	Explain the definition and the properties of special functions and the linearity, boundedness and convergence of fractional differential and integral operators.	Remember
CO 2	Understand the significance of Mittag-Leffler (ML) function, the existence and uniqueness of linear fractional differential equation.	Understand
CO 3	Apply the Laplace transformation to solve Cauchy problem with Caputo fractional derivative.	Apply
CO 4	Analyze the importance of fractional derivative and fractional integral in real time applications. Demonstrate numerical analysis on ML function, fractional derivative and fractional differential equation	Analyze
CO 5	Study the physical and biological models with fractional derivative rigorously to get feasible and accurate solution.	Skill

b. Syllabus

Units	**Content**	**Hrs.**
I	Fractional Calculus**:** History and Motivation, Special functions in Fractional Calculus**:** Gamma Function, Beta function, Mittag Leffler (ML) Function, Properties of ML function, Laplace Transform of Mittag-Leffler Function. Fractional Derivatives and Integrals**:** Grunwald Letnikov Fractional Derivative, Riemann Liouville (RL) Fractional Derivative, Caputo Fractional Derivative and Riemann Liouville Fractional Integral	15
II	Analysis on Fractional Operators**:** Properties of RL Fractional Integral and Derivative, Caputo Fractional Derivative (Convergence, Boundedness), Relation in between RL and Caputo Fractional Derivative, Laplace Transform of Fractional Operators**. (**	15
III	Linear Fractional Differential Equations (FDE)**:** Existence and Uniqueness Theorem, Successive approximation, Laplace Transform method for FDE in Homogeneous and Non homogeneous case	15
IV	Applications of FDE**:** Physical models, biological models, Fractional Inverse Problems. Numerical Analysis**:** ML function, Approximation of Fractional operators, Numerical solution of FDE	15
	References: **1.** K.Deithelm, The Analysis of Fractional Differential Equations, Springer, Berlin, 2010**.** **2.** K.S.Miller and B**.** Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley,	

	New York, 1993. **3.** K. B. Oldham and J.Spanier, The Fractional Calculus, Academic Press, New York, 1974. **4.** I.Podlubny,Fractional Differential Equations, Academic Press,New York,1999. **5.** S. G.Samko, A. A.Kilbas and O. I.Marichev, Fractional Integrals and Deriva-tives (Theory and Applications), Gordon and Breach Science Publishers, Amsterdam, 1993.	

c. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	1	1	3	1
CO2	3	2	2	3	1
CO3	3	2	2	3	1
CO4	3	2	3	3	3
CO5	3	3	3	3	3

d. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Assignments	2	2	-	-	2
Seminar	-	-	2	2	-
Test	5	5	5	5	5
Attendance	1	1	1	1	1
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **(Objective - 10 x 1 = 10 marks)**	2	2	2	2	2
Part – B **(Short Answer - 5 x 4 = 20 marks)**	10	10	-	-	-
Part – C **(Essay- 3 x 10 = 30 marks)**	-	-	10	10	10
Total	**12**	**12**	**12**	**12**	**12**

Model Question Paper

Sl. No.	Model Questions	Specification	Level
	Part – A: Objective Type **Multiple choice 10 x 1 = 10**		
1	The Gamma function converges in the A. right half plane C. left half plane B. real line D. whole complex plane	Recognize	Remember
2	The gamma function $\Gamma(n)$ has simple poles at A. $n \in \mathbb{N} \cup \{0\}$ B. $n \in \mathbb{N}^-$ C. $n \in \mathbb{N}^+ \cup \{0\}$ D. None of the above	Recall	Remember
3	Beta function $B(\alpha, \beta)$ will lose its analyticity for A. $Re(\alpha) > 0,\ Re(\beta) > 0$ C. $Re(\alpha) < 0,\ Re(\beta) < 0$ B. $Re(\alpha) = 0,\ Re(\beta) = 0$ D. Choices B) and C)	Recognize	Remember
4	Pick out the wrong choice on Mittag Leffler (ML) function A. MittagLeffler function is an entire function. B. MittagLeffler function is an unbounded function. C. It is a smooth function. D. it has no semigroup property.	Recognize	Remember
5	Pick the correct answer. For $\alpha > 0, let\ I^{\alpha}$f denotes the Riemann Liouville fractional integral, $f \in L^p([a,b])$. Then $L^p \ni f \to I^{\alpha}$f $\in X$ $A. L^p([a,b])$ B. $C([a,b])$. C. No such X space D. A) and B)	Recognize	Remember
6	For 0< α<1, $\lambda > 0\ and\ x > 0$, Evaluate $I^{\alpha}e^{\lambda x}$. A. $I^{\alpha}e^{\lambda x} \neq \frac{e^{\lambda x}}{\lambda^{\alpha}}$ C. $I^{\alpha}e^{\lambda x} = \frac{e^{\lambda x}}{\lambda^{\alpha}}$ B. $I^{\alpha}e^{\lambda x} = \frac{e^{\lambda x}}{\lambda}$ D. $I^{\alpha}e^{\lambda x} = \frac{e^{\lambda x}}{\lambda^{2}}$	Recognize	Remember
7	Evaluate the relation $E_{\alpha}(z) + E_{\alpha}(z -)\ , z \in \mathbb{C}$ A. $2E_{2\alpha}(z^2)$ B. $2E_{\alpha}(z^2)$ C. $E_{2\alpha}(z^2)$ D. $2E_{2\alpha}(z)$	Recall	Remember
8	Identify the incorrect answer. A. Riemann Liouville fractional derivative of a constant is zero. B. Caputo fractional derivative of a constant is zero. C. Caputo fractional derivative is a linear operator. D. Riemann Liouville fractional derivative relates with Caputo fractional derivative.	Identify	Remember

9	Evaluate $\int_0^t s^{\alpha-1}E_{\alpha,\alpha}(-\lambda s^{\alpha})ds$. A. $\frac{1}{\lambda}(1-E_{\alpha}(-\lambda t^{\alpha})$ $C.\frac{1}{\lambda}(1+E_{\alpha}(-\lambda t^{\alpha})$ B. $\frac{1}{\lambda}(1-E_{\alpha}(\lambda t^{\alpha})$ $D.\frac{1}{\lambda}(1+E_{\alpha}(\lambda t^{\alpha})$	Evaluate	Understand
10	Evaluate theLaplace transform of $E_1(t^2)$ A. Does not exist B. $\frac{1}{s}$ C. $\frac{1}{\sqrt{s}}$ D. $\frac{1}{s-2}$	Identify	Apply
	PART – B Short Answer **The answer should not exceed 200 words 5 x 4 = 20**		
11	a) Explain the significance of fractional differential equation via stability analysis. (or) b) Demonstrate the numerical approximation of fractional operators.	Explain	Analyse
12	a) Discuss the convergence of fractional model to classical model. (or) b) Define Mittag Leffler function and its properties.	Discuss Define	Understand
13	a) Give two real-life examples using fractional derivative of order $0 < \alpha \leq 1$(or) b) Give two real-life examples using fractional derivative of order $1 < \alpha \leq 2$	Cite Examples	Analyse
14	a) Illustrate the existence of Caputo fractional derivative of the function $f(x) = \sqrt{x-a}$ at $x = a$. (or) b) Solve the linear initial value problem with Caputo fractional derivative by means of Laplace transformation $d^{0.5}\,y(t) + 2y(t) = \sin t\,, t\,\epsilon\,(0,5).$ $y(0) = 10.$	Illustrate	Apply
PART – C Essay Answer **The answer should not exceed 400 words 3 x 10 = 30**			
15	a) Derive the equivalent integral form of the nonlinear fractional initial value problem. Describe the non-local property of the Caputo fractional derivative using the solution representation. (or) b) Picard s successive approximation for fractional nonlinear initial value problem with Caputo fractional derivative.	Describe	Analyse
16	a) Geometrical interpretation of fractional integral operator and Numerical representation of ML function. (or) b) Find the Riemann Liouville fractional derivative of $f(t) = 1\ , t \geq 0$ and find the error estimate with its numerical approximation.	Explain Discuss	Analyse
17	a) Describe the fractional harmonic oscillator model in detail. (or) b) Explain fractional heat conduction problem and solve it.	Assess	Skill

SEMESTER - IV					
Course Code	**Course Name**	**L**	**T**	**P**	**Credits**
SAM04P	Project work	4	-	-	4

a. Course Outcome (CO)

On the successful completion of the course, the student will be able to

	Course Outcome	Level
CO 1	Recall all techniques and models covered in this domain.	Remember
CO 2	Convert a real life problem into a statistical or mathematical problem.	Understand
CO 3	Apply various statistical and mathematical tools to solve the problem.	Apply
CO 4	Write a scientific report.	Analyze
CO 5	Present a scientific report.	Skill

b. Mapping of Program Outcomes with Course Outcomes

	PSO1	PSO2	PSO3	PSO4	PSO5
CO1	3	1	2	1	1
CO2	2	2	2	2	1
CO3	1	2	3	3	1
CO4	1	1	2	3	2
CO5	0	0	0	3	3

c. Evaluation Scheme

	CO1	CO2	CO3	CO4	CO5	Total
Internal	8	8	8	8	8	40
External	12	12	12	12	12	60
Total	**20**	**20**	**20**	**20**	**20**	**100**

e. Mapping Course Outcome with Internal Assessment (40 Marks)

	CO1	CO2	CO3	CO4	CO5
Seminar	6	6	6	6	6
Attendance	2	2	2	2	2
Total	8	8	8	8	8

f. Mapping Course Outcome with External Assessment (60 Marks)

Category	CO1	CO2	CO3	CO4	CO5
Part – A **Project Report**	8	8	8	8	8
Part – B **Presentation**	4	4	4	4	4
Total	**12**	**12**	**12**	**12**	**12**