Semester I

Subject Code: MAT011 **Credits: 3**

Mathematics I

1. Inequalities, Sequences: Bounded and Unbounded Sequences, Convergent Sequences, Series, Tests of Convergence (without proofs).

2. Functions On Subsets of $\mathbb{R}$, Limits and Continuity, Intermediate Value Theorem (without proof)

3. Review of Differential Calculus; Formal Definition of Differentiability, Applications to : Rate of Change, Tangents to Plane Curves and motion on a Straight Line; Roll's and Mean Value Theorems with Geometric Illustrations (without proofs), Increasing and Decreasing Functions, Higher Order Derivatives, Local Maxima and Minima (without proofs);
   Curves: Curvature , Tangent Vectors to Space Curves.

4. Review of Integral Calculus, Area Under Curves, Integration as the Limit of Sums, Fundamental Theorem of Integral Calculus (without proof), Bernouli's Formula, Applications to Area, Arc Length, Surface Area and Volumes of Surface of Revolution.

5. Partial Differentiation, Formal chain rules; Surfaces: Normal to Surfaces.

References:

1. G.B. Thomas and R.L. Finney, Calculus and Analytic Geometry, 9th International Edition, Addison Wesley, 2002.

Semester II

Subject Code: MAT021 **Credits: 3**

Mathematics II

1. Double Integration; Change of Order
2. Vector Calculus: Gradient, Curl, Divergence, Line and Surface and Volume Integrals, Green's theorem, Stoke's theorem and Gauss Divergence theorem
3. Ordinary Differential Equations : Separable Equations, Linear Equations, Homogeneous and Exact Equations Basic Examples and Methods of Solutions;

References

1. G.B. Thomas and R.L. Finney, Calculus and Analytic Geometry, 9th International Edition, Addison Wesley, 2002.
2. E. Kreyszig, Advanced Engineering Mathematics, 8th International Edition, John Wiley and Sons, 1999.

Subject Code: MAT022 **Credits: 2**

Basic computing lab

1. Unit I: Libre Office Functions, Generation of sequences and partial sums, matrix operations
2. Unit II: Solving systems of linear equations using Libre Office, Numerical Solution of nonlinear equations using Newton-Raphson
3. Unit III: Solution of IVP for ODE using Euler and Runge-Kutta of second order
4. Unit IV: Numerical Integration
5. Unit V: Interpolation

Reference

1. S.D. Conte and C. de Boor, Elementary Numerical Analysis an algorithmic approach, , 3rd edition, McGraw-Hill International editions, 1988.

Semester III

Subject Code: MAT031 **Credits: 4**

Mathematics III

1. Partial Differential Equations: Introduction and Methods for First order PDEs, Second and Higher Order Equations With Constant Coefficients
2. Fourier Series, Half Range Series, Applications to Boundary Value Problems - Vibration of Strings, One dimensional Heat Equation, Steady State two dimensional Heat Equation.
3. Fourier Transforms and Applications to PDE.
4. Matrices, Determinants, Eigenvalues and Eigenvectors, Cayley-Hamilton theorem, Definitions and Examples of Group Theory and Vector Spaces, Matrices as Linear Maps.

References

1. E. Kreyszig, Advanced Engineering Mathematics, 8th International Edition, John Wiley and Sons, 1999.
2. I.N. Sneddon, Elements of Partial Differential Equations, McGraw-Hill Book Company, Inc., New York, 1957.
3. S. Kumaresan, Linear Algebra: A Geometric Approach,PHI Learning, 2000.
4. M. Braun, Differential Equations and Their Applications, An Introduction to Applied Mathematics, Fourth Edition, Springer, 1999.
5. M. Artin, Algebra, Prentice-Hall Of India Pvt. Limited, 1996.

Semester IV

Subject Code: MAT041 **Credits: 4**

Applied Statistics: Theory

1. Motivation Examples of applications of Statistics in Physical and Biological Sciences; Uses and misuses of Statistics.

2. Descriptive Statistics Scientific investigation; Population and sample; Data collection; Types of variables and scales of measurement; Methods of displaying data - graphical and tabular methods; Grouped data - Frequency distributions, histograms and frequency polygons; Measures of central tendency - Mean, Median and Mode; Quantiles - Quartiles, percentiles; Measures of dispersion - Range, variance and standard deviation; Measures of skewness and kurtosis; Exploratory data analysis - stem and leaf diagram and box plot. Bivariate data; scatter plot; Covariance and Correlation coefficient. Applications.

3. Probability Random experiment; Sample point, Event and Probability; Rules of Probability; Conditional Probability; Independence of Events; Bayes' Rule. Applications

4. . Discrete Random variables Definition; sum and linear composite of random variables; Mean and variance; Bernoulli, Binomial, geometric and negative binomial distributions; hypergeometric distribution; Poisson distribution. Applications [5]

5. Continuous Random Variables Definition; Uniform and exponential distributions; Normal distribution and its properties; Standard normal distribution; Transformation from a general normal distribution to standard normal; Checking for normality of data; Applications; [6]

6. Joint distributions and independence Joint distributions; covariance and correlation; uncorrelatedness and independence; Transformations; Chi-square, t and F distributions as sampling distributions from a normal distribution; Central limit Theorem (Statement only); Applications.

7. Point Estimation and Confidence Intervals Point estimation of the population mean and standard deviation of a normal distribution; Estimation of proportion; Confidence intervals; Large sample methods; Applications. [5]

8. Hypothesis Testing Hypothesis - simple and composite; Null and alternative; Test of Hypothesis; Type I and Type II errors; Level and power of a test; p-value; Tests for mean and standard deviation; Test for proportion; one tail or two tails. Applications. [5]

9. Comparison of two populations Paired-observation comparisons; Difference between (a) population means and (b) population proportions using independent random samples; Equality of population variances; Large sample tests; Applications [4]

10. Designed Experiments Completely randomized, randomized complete blocks and Latin square experiments; Analysis of variance; Introduction to multifactor experiments; Applications. [6]

11. Regression Analysis Simple linear regression model; Estimation of parameters; Method of Least squares; Correlation coefficient as a measure of goodness of fit; Multiple linear regression; Inference on parameters; R2 and adjusted R2; Applications. [6]

References:

1. M.L. Samuels, and J.A. Witmer , Statistics for the life sciences, 3rd Edition, Prentice Hall, 2003.

2. H.E. Van Emden, Statistics for terrified Biologists, Blackwell Publishing, 2008.

3. P. Fornacini, The uncertainty in physical measurements - An introduction to Data Analysis in the Physics Laboratory, Springer, 2008.

4. R. Barlow, Statistics - A guide to the use of statistical methods in the Physical Sciences, Wiley, 1999.

5. J.N. Miller, and J.C. Miller Statistics and Chemometrics for Analytical Chemistry, 5th Edition, Pearson Education, 2005.

6. A.D. Aczel, and J. Sounderpandian Complete Business Statistics, 7th Edition, McGraw-Hill, Irwin, 2008.

Subject Code: MAT042 **Credits: 2**

Scientific Computing Lab

1. Methods of displaying data - graphical and tabular methods; Grouped data - Frequency distributions, histograms and frequency polygons;

2. Measures of central tendency - Mean, Median and Mode;

3. Quantiles - Quartiles, percentiles; Measures of dispersion - Range, variance and standard deviation;

4. Measures of skewness and kurtosis; Exploratory data analysis - stem and leaf diagram and box plot.

5. Bivariate data; scatter plot; Covariance and Correlation coefficient.

References

1. S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, Sultan Chand & Sons, 2007.

2. R.V. Hogg and A.T. Craig, Introduction to Mathematical Statistics, 5th Edition, Pearson Education, 1995.

3. K.M. Ramachandran and C.P. Tsoko, Mathematical Statistics with Applications in R, 2nd Edition, Academic Press, 2014.

4. J.D. Miller, Statistics for Data Science, Packt Publishing Ltd., 2017.

Semester V

Subject Code: MAT051 **Credits: 4**

Analysis I

1. **Basics of real number system:** upper and lower bounds, LUB and GLB, LUB axiom, Archimedean property, density of rationals and irrational, nested interval theorem.

2. **Metric Spaces:** open set, closed set, interior, closure, perfect set, compact set, connected set.

3. **Sequences in $\mathbb{R}$ and $\mathbb{C}$ and their convergence:** algebra of convergent sequences, monotone sequences, Cauchy sequences and their convergence, subsequences, Bolzano-Weierstrass theorem, recursively defined sequences.

4. **Continuity:** sequential definition, ϵ-δ definition and their equivalence. Intermediate value theorem and Weierstrass theorem on continuous functions on a closed and bounded interval. Monotone functions. **Uniform continuity.**

5. **Limits:** All kinds: both sided limits, left and right limits, meaning of $\lim_{x\to a} f(x) = \infty$, $\lim_{x\to\infty} f(x) = \ell$, $\lim_{x\to\infty} f(x) = \infty$ etc. continuity of f and the limit f at a.

6. **Differentiation:** characterization of differentiability: f is differentiable at a iff there exists a functions f_1 continuous at a such that $f(x) = f(a) + f_1(x)(x - a)$. Derivation of algebra of differentiable functions, chain rule using this characterization, Rolle's theorem, mean value theorem, lots of applications of mean value theorem including theoretical ones such as constancy of functions (with an interval as its domain) whose derivative is zero, sign of the derivative and monotonicity of functions,inequalities; Taylor's theorem. and its application to approximation and theory of local extrema; convex functions.

References

1. R.G. Bartle and D.R. Sherbert, Introduction to Real Analysis, 3rd Edtion, Wiley International Student edition, 2001.

2. A. Kumar, S.Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

3. K.A. Ross, Elementary Analysis: The theory of Calculus, Springer International Edition, Indian Reprint, New Delhi, 2004.

4. R.S. Strichartz, The Way of Analysis, Revised Edition, Jones and Bartlett Publishers, 2000.

5. T. Apostol, Mathematical Analysis, 2nd Edtion, Narosa Publishing House, 1985.

6. W. Rudin, Principles of Mathematical Analysis, Wiley International Edition, 1985.

Subject Code: MAT052 **Credits: 4**

Linear Algebra I

1. Systems of linear equations, Gaussian elimination,Geometric interpretation of solution sets, non-homogeneous systems of linear equations and the associated homogeneous systems and relation between their solution sets; Existence of nontrivial solutions for a homogeneous system of m linear equations in n variables with $m < n$. Matrix notation, row-reductions etc may be employed.

2. Vector spaces: definition and lots of examples, vector subspaces, vector subspaces generated by a set of vectors, Linear dependence and independence, basis, dimension formula for the sum a two vector subspaces.

3. Linear Transformations: Definition and examples, Kernel and image, Rank-nullity theorem, Matrix Representation w.r.t ordered bases.

4. Inner product spaces: Cauchy-Schwarz inequality, (unoriented) angle between two nonzero vectors,orthogonality,orthogonal and orthonormal basis, orthogonal decomposition, orthogonal transformation, symmetric and skew-symmetric transformations, the associated matrices are of special kind. Rotations in $\mathbb{R}^2$, $\mathbb{R}^3$. Reflections. Distance preserving maps on a finite dimensional inner product space.

5. Determinants: As a skew-symmetric multilinear function; Laplace expansion, determinant of a product; use of determinants for area and orientation (basic notions)

6. Diagonalization: Classification of conics; eigenvalues and eigenvectors; characteristic equation, characteristic roots versus eigenvalues, spectral theorem for a symmetric operator in a f.d. inner product space.

References.

1. S. Kumaresan, Linear Algebra–A Geometric Approach, 12th reprint, Prentice Hall of India, 2011.

2. G. Strang, Linear Algebra and its applications, 8th Indian reprint Indian edition, Cengage Learning, 2011.

3. S.H. Friedberg and A.J.Insel, L.E. Spence, Linear Algebra, 4th edtion, Prentice-Hall of India, 2003.

4. K. Hoffman and R. Kunze, Linear Algebra, 2nd edtion, Prentice Hall of India, 2003

Subject Code: MAT053 **Credits: 4**

Discrete Mathematics

1. Logic Statements, Truth Tables, Connectives, Normal forms
2. Boolean Algebra Review of Relations, Partial Ordering, Posets, Hasse Diagram, Lattices, Boolean Algebra, Definition of a Boolean Algebra as An Algebraic System and the Equivalence of the algebraic definition and lattice theoretic definition, Boolean Expressions and Karnaugh Maps.
3. Counting Permutations and Combinations, Binomial Theorem, Counting Objects with Repetitions, General Form of Binomial Theorem (without proof), Principle of Inclusion and Exclusion, Pigeonhole Principle, Combinatorial Representation of permutations, tree representations, descents and Eulerian polynomials
4. Graphs Introduction to Graphs, Graph Isomorphism, Connectivity, Euler and Hamilton Paths
5. Finite Automata and Turing Machines Regular Languages and Finite Automata, Context Free Languages and Pushdown Automata, Context Sensitive Language and Linear Bounded Automata, Turing Machines and Recursively enumerable languages

References:

1. K. Rosen, Discrete Mathematics and its Applications, 5th Edition, Tata McGraw-Hill Publishing, 2015.
2. R.P. Girimaldi, Discrete and combinatorial mathematics: An applied introduction, 5th Edition, Pearson, 2003.
3. J.E. Hopcroft, R. Motwani, J.D. Ullman, Introduction to Automata Theory, Languages, and Computation, 3rd edition, Addison Wesley, 2007.
4. . T.Sengadir, Discrete Mathematics and Combinatorics, Pearson,2009.

Subject Code: MAT054 **Credits: 4**

Algebra I

1. Groups: starting as a group of transformations; definition of an abstract group; examples: finite, infinite, abelian, nonabelian; lots of examples including matrix groups, $ax + b$ group, Euclidean motion group, the group of all nth roots of unity for $n \in \mathbb{N}$.

2. Subgroups: existence of smallest subgroups of a group G containing a subset $S \subset G$; cyclic subgroups; cyclic groups, order of an element; Cosets of subgroups; Lagrange's theorem.

3. Homomorphisms of groups, kernel, image, normal subgroups;

4. Group actions: definition and lots of concrete examples such as $GL(n, \mathbb{R})$ action on $\mathbb{R}^n$, $O(n)$ action on S^{n-1}, $SL(2, \mathbb{R})$ action on upper half-plane, along with standard examples such a right and left actions, conjugate action etc. orbits and stabilizers; orbit formula; applications: Lagrange's theorem, $\mathbf{Z}_2$ action on a finite group; class equation etc. Transitive actions.

5. Special classes of groups: Dihedral groups, Groups of units in $\mathbf{Z}_n$, Symmetric groups and alternating groups.

6. Quotient groups and homomorphism theorems.

7. Sylow's theorem: Cauchy's theorem and Sylow's theorem via group actions; and classification of groups of order up to 12, applications of Sylow's theorem, Direct product, Finite abelian groups, Structure theorem.

References.

1. J.B. Fraleigh, A First course in Abstract Algebra, 7th edtion, Pearson Education, 2003.

2. J.A. Gallian, Contemporary Abstract Algebra, 4th Edition, Narosa Publishing House, 2007.

3. P.B. Garrett. Abstract Algebra, Chapman & Hall/CRC Press, Special Indian Edtion, 2009.

4. M. Artin, Algebra, Prentice-Hall of India, 1994.

5. C. Lanski, Concepts in Abstract Algebra, American Math. Society, Indian Edtion, by Universities Press, 2010.

6. I.N. Herstein, Topics in Algebra, 2nd Edition, John-Wiley & Sons, 1975.

Semsester VI

Subject Code: MAT061 **Credits: 4**

Analysis II

1. Riemann-Darboux integral: Riemann-Darboux integrability, integrability of continuous functions, monotone functions, functions with a finite number of points of discontinuity. Definition of Riemann integral; a bounded function on $[a, b]$ is Riemann-Darboux integrable iff it is Riemann integrable, proof may be indicated at least for the class of continuous functions, application to summation of series. Mean value theorems for integrals. Fundamental theorem of calculus, Integration by parts.

2. Infinite series: of real/complex numbers, series of positive terms, geometric series, comparison test, absolute convergence and conditional convergence; ratio, root tests, integral test, Alternating series test; Summation by parts and Abel's formula; Dirichlet's test; Riemann's theorem on a conditionally convergent series.

3. Pointwise and uniform convergence: Lots of examples; Uniformly Cauchy; Continuity, differentiation and integrability of the uniform limits in the appropriate classes of functions;

4. Series of functions and Power series: Weierstrass M-test, Dini's theorem; Complex power series and radius of convergence. Term-wise differentiability of the power series in its disk of convergence.

5. Theory of Fourier series (optional).

References

1. R.G. Bartle and D.R. Sherbert, Introduction to Real Analysis, 3rd Edtion, Wiley International Student edition, 2001.

2. K.A. Ross, Elementary Analysis: The theory of Calculus, Springer International Edition, Indian Reprint, 2004.

3. E.D. Gaughan, Introduction to Analysis, 5th Edtion,Indian reprint by Universities Press, New Delhi, 2010.

4. T. Apostol, Mathematical Analysis, 2nd Edtion, Narosa Publishing House, 1985.

5. W. Rudin, Principles of Mathematical Analysis, Wiley International Edition, 1985.

Subject Code: MAT062 **Credits: 4**

Algebra II

1. Solvable groups. Nilpotent groups.

2. Definition and examples of rings, Commutative rings, Fields, subrings. homomorphism of rings. Kernel and image of a homomorphism. Characteristic of a ring. Ideals (two sided). Quotient rings. Prime ideals, maximal ideals and their characterization. Polynomial rings. Divisibility. units. Factorization in a ring. Irreducible and prime elements in a ring. Unique factorization domain, principal ideal domain and euclidean domains.

3. Fields. Field extensions. Finite fields. Finite and algebraic extensions. Classical geometric constructions. Galois theory- fundamental theorem of Galois theory and Abel's theorem, Fundamental Theorem of Algebra.

References:

1. J.A. Gallian, Contemporary abstract algebra, Narosa Publishers, India, 1999.

2. N. Jacobson, Basic Algebra I, Second Edition, Dover, 2009.

3. O. Zariski, P. Samuel, Commutative algebra I, Springer, 1975.

4. J.B. Fraleigh, A First Course in Abstract Algebra, Narosa Publishers, India, 2003.

5. M. Artin, Algebra, Prentice Hall India, 1996.

6. J. Rotman, Galois Theory, Universitext, Springer, 1998.

Subject Code: MAT063 **Credits: 4**

Ordinary Differential Equations I

1. **First order differential equations:** Introduction, Separable equations, Exact equations: Integrating factors, Existence and uniqueness theorem.

2. **Second order differential equations:** Linear equations with constant coefficients, Non-homogeneous equations, Method of variation of parameters, Method of undetermined coefficients, Power series method, Singular points, regular singular points. Euler's equation, Frobenius method, Laplace transformation and its application. Higher order equations.

3. **System of differential equations:** Linear homogeneous and non-homogeneous system of equations, Linear algebraic method to find solutions to homogeneous equations, Method of variation of parameters and Laplace transformation for solving non-homogeneous equations.

4. **Qualitative theory:** Stability of linear system of ODEs, Stability of points of equilibrium, Phase space and the solutions as orbits in the phase space and then qualitative properties of solutions, Predator-prey problems.

5. Separation of variables and Fourier series method, Sturm-Liouville method.

References

1. M. Braun, Differential Equations and their applications, Fourth Edition, Springer, 1993.

2. S.L. Ross, Differential Equation, Fourth Edition, John Wiley & Sons, 1984.

3. A.K. Nandakumaran, P.S. Datti and R.K. George, Ordinary Differential Equations: Principles and Applications, Cambridge University Press, 2017.

4. T. Myint-U, Ordinary Differential Equations, Elsevier, North-Holland, 1978.

5. G.F. Simmons & S.G. Krantz, Differential Equations: Theory, Technique, and Practice, Tata Mc-Graw Hill, 2012.

6. E.A. Coddington, An Introduction to Ordinary Differential Equations, Dover, 1961.

7. L. Perko, Differential Equations and Dynamical Systems, Third Edition, Springer, 2006.

8. M.W. Hirsch, S. Smale, R.L. Devaney, Differential Equations, Dynamical Systems, and an Introduction to Chaos, Academic Press.

Subject Code: MAT064 **Credits: 3**

Numerical Analysis

1. Solution of Equations, Linear Systems and Algebraic Eigenvalue Problems Solution of algebraic and transcendental equations: Fixed-point iteration method, Newton's method; Linear system (Direct methods): Gaussian elimination - Pivoting - LU Decomposition; Vector and Matrix norms - Error Analysis and Condition numbers; Linear system (Iterative methods): Gauss-Jacobi and Gauss-Seidel - Convergence considerations; Eigenvalue problem: Power method - Jacobi for a real symmetric matrix.

2. Interpolation, Differentiation and Integration

   Interpolation: Lagrange's interpolation - Errors in Lagrange's interpolation - Newton's divided differences - Newton's finite difference interpolation - Optimal points for interpolation - Piecewise Interpolation: Piecewise linear and piecewise Cubic Spline interpolation

   Numerical differentiation: Numerical differentiation based on interpolation, finite differences, method of undetermined coefficients;

   Numerical integration: Newton Cotes formulae - Gaussian quadrature - Errors in Simpson's rule and Gaussian quadrature -method of undetermined coefficients -quadrature rules for Multiple integrals.

3. Ordinary Differential Equations Single-Step methods: Euler's method and Modified Euler's method -Taylor series method - Runge-Kutta method of fourth order - Multistep methods: Adams-Bashforth - Adams - Moulton methods - Stability considerations - Two point BVPs: Finite Difference method - Linear problems with Dirichlet and derivative boundary conditions - Stiff equations - Eigenvalue problems.

References

1. K.E. Atkinson, An Introduction to Numerical Analysis, Wiley, 1989

2. G.M Phillips, P.J. Taylor, Theory and Applications of Numerical Analysis, Second Edition, Elsevier, New Delhi, 2006.

3. L.B. Richard, J.D. Faires, Numerical Analysis, Nineth Edition, Brooks/Cole, Cengage Learning, 2011.

4. E. Isaacson, H.B. Keller, Analysis of Numerical Methods, Dover Publication, 1994.

5. A. Quarteroni, F. Saleri and P. Gervasio, Scientific computing with MATLAB and Octave, Springer, 2006.

6. S.D. Conte, C. de Boor, Elementary Numerical Analysis, Third Edition, McGraw-Hill Book Company, 1983.

7. D. Kincaid, W. Chenney, Numerical Analysis: Mathematics of Scientific Computing, Brooks/Cole Pub. 2nd Edition, 2002.

8. B. Bradie, A Friendly Introduction to Numerical Analysis, First Edition, Pearson Education, New Delhi, 2007.

Subject Code: MAT065 **Credits: 2**

Numerical Analysis Lab

1. Solution of Equations, Linear Systems and Algebraic Eigenvalue Problems Solution of algebraic and transcendental equations: Fixed-point iteration method, Newton's method; Linear system (Direct methods): Gaussian elimination - Pivoting - LU Decomposition; Vector and Matrix norms - Error Analysis and Condition numbers; Linear system (Iterative methods): Gauss-Jacobi and Gauss-Seidel - Convergence considerations; Eigenvalue problem: Power method - Jacobi for a real symmetric matrix.

2. Interpolation, Differentiation and Integration

   Interpolation: Lagrange's interpolation - Errors in Lagrange's interpolation - Newton's divided differences - Newton's finite difference interpolation - Optimal points for interpolation - Piecewise Interpolation: Piecewise linear and piecewise Cubic Spline interpolation

   Numerical differentiation: Numerical differentiation based on interpolation, finite differences, method of undetermined coefficients;

   Numerical integration: Newton Cotes formulae - Gaussian quadrature - Errors in Simpson's rule and Gaussian quadrature -method of undetermined coefficients -quadrature rules for Multiple integrals.

3. Ordinary Differential Equations Single-Step methods: Euler's method and Modified Euler's method -Taylor series method - Runge-Kutta method of fourth order - Multistep methods: Adams-Bashforth - Adams - Moulton methods - Stability considerations - Two point BVPs: Finite Difference method - Linear problems with Dirichlet and derivative boundary conditions - Stiff equations - Eigenvalue problems.

References

1. K.E. Atkinson, An Introduction to Numerical Analysis, Wiley, 1989

2. G.M Phillips, P.J. Taylor, Theory and Applications of Numerical Analysis, Second Edition, Elsevier, New Delhi, 2006.

3. L.B. Richard, J.D. Faires, Numerical Analysis, Nineth Edition, Brooks/Cole, Cengage Learning, 2011.

4. E. Isaacson, H.B. Keller, Analysis of Numerical Methods, Dover Publication, 1994.

5. A. Quarteroni, F. Saleri and P. Gervasio, Scientific computing with MATLAB and Octave, Springer, 2006.

6. S.D. Conte, C. de Boor, Elementary Numerical Analysis, Third Edition, McGraw-Hill Book Company, 1983.

7. D. Kincaid, W. Chenney, Numerical Analysis: Mathematics of Scientific Computing, Brooks/Cole Pub. 2nd Edition, 2002.

8. B. Bradie, A Friendly Introduction to Numerical Analysis, First Edition, Pearson Education, New Delhi, 2007.

Semester VII

Subject Code: MAT071 **Credits: 4**

Linear Algebra II

1. Structure of a single linear transformation: Invariant subspaces, semi-simple and nilpotent maps; Characteristic polynomial and minimal polynomial; Primary decomposition, cyclic subspaces and decomposition into cyclic subspaces, Jordan and rational canonical forms.

   Spectral theorem for orthogonal and unitary linear maps. Singular value decomposition.

2. Modules: Definition and examples, submodules, direct sums, module homomorphisms, quotient modules, torsion elements, free modules

3. Structure theorem for finitely generated modules over a PID, applications to a single linear transformation and finitely generated abelian groups. Cayley-Hamilton theorem using canonical forms.

4. Smith normal form of a matrix over a Euclidean domain and applications.

References

1. K. Hoffman and R. Kunze, Linear Algebra, 2nd edition, Prentice-Hall of India, 2003.

2. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul, Basic abstract algebra, 2nd edition, Cambridge University Press, Indian edition by Foundation Books, 1995.

3. D.S. Dummit and R.M. Foote, Abstract Algebra, 2nd edtion, Indian Edtion by Pearson, 2005.

Subject Code: MAT072 **Credits: 4**

Topology

1. A quick introduction to finite, countable and uncountable sets.
2. A quick review of Metric spaces, open sets, continuity and convergence of sequences.
3. Topological spaces: Metric topology, indiscrete, discrete, co-finite, co-countable, VIP and outcast topologies. Continuity of maps between spaces; Equivalence of the definitions of continuity of a function between topological spaces and ϵ-δ definition in case of metric spaces. Algebra/lattice of continuous real valued functions on a topological space; examples of absence of non-constant continuous functions in contrast with the abundance of continuous functions on a metric space. Existence of a smallest topology containing a given class of subsets of a set, Finer/coarser topologies.
4. Homeomorphisms: Lots of examples.
5. Basis for a topological space and basis for some topology on a set. Order topology and lower limit topology. Continuity in terms of bases.
6. Closed sets, continuity in terms of closed sets, closures, limit points, cluster (accumulation) points, every point of an open set in a Euclidean/normed linear space is a cluster point. Interior and boundary of a set.
7. Hausdorff spaces, countability axioms, separability. Sequences and their convergence
8. New topologies from the old: (i) Given a collection $\mathcal{A} \subset P(X)$ the description of the smallest topology containing $\mathcal{A}$. (ii) Subspace topology (iii) Product topology and (iv) quotient topology
9. Compactness and local compactness. Characterization of compact subsets a metric space.
10. Connectedness and path-connectedness; Local connected spaces and locally path-connected spaces.
11. Separation of a point and a closed set and two disjoint closed sets. Normal spaces, Locally compact Hausdorff spaces, one-point compactification. Uryosohn's lemma and Tietze extension theorem, Urysohn's lemma, Urysohn Metrization Theorem, Tycknoff's theorem, Stone-Čech Compactification.

References

1. J.R. Munkres, Topology, 2nd Edition,Prentice Hall of India, 2000.

2. M.A. Armstong, Basic Topology, Springer International Edition, 2005.

3. G.E. Bredon, Topology and Geometry, Springer International Edition, 2008.

4. S. Kumaresan, Topology of Metric Spaces, 2nd Edition, Narosa Publishing, 2011.

Subject Code: MAT073 **Credits: 4**

Analysis III (Measure Theory and Integration)

1. A quick review (in about 2 lectures) of Riemann integral.

2. Outer measure and Measure in $\mathbb{R}$ and $\mathbb{R}^n$. Definition of outer measure of a subset of $\mathbb{R}^n$ and its properties; Cantor (ternary) set. Caratheodary's definition of a (Lebesgue) measurable set. The sigma-algebra of (Lebesgue) measurable sets. The inner and outer regularity of (Lebesgue) measurable sets. Construction of non-measurable sets.

3. Abstract measure theory: σ-algebra $\mathcal{B}$ of subsets of a set X, Measurable space as $\mathcal{M}$, a (positive) measure on a measurable space, examples such as Lebesgue measure, counting measure, Dirac measure at a point, Borel-Cantelli Lemma

4. Measurable functions: on an abstract measure space. on Euclidean spaces, standard results on the class of measurable functions; any non-negative measurable function is the limit of an increasing sequence of simple functions; continuous, monotone, derivative of differentiable functions as examples on $\mathbb{R}$, cantor's function.

5. Lebesgue integral: Integral of a simple function, non-negative measurable functions, any real values measurable function, complex valued functions on a measure space $\mathcal{M}$.

   The map $E \mapsto \int_E f$ for a fixed non-negative simple function (respectively a non-negative measurable function) is a measure on $(X, \mathcal{B})$. Monotone convergence theorem, Fatou's lemma, Dominated and bounded convergence theorems.

6. Signed measure, Hahn decomposition, Jordan decomposition, Radon-Nykodim theorem, Fubini's Theorem.

7. L^p spaces, Riesz-Fischer theorem. Concrete classes of functions which are in L^p and not in L^p. Density of simple functions, continuous functions and smooth functions with compact support in L^p spaces over $\mathbb{R}$ for $1 \leq p < \infty$.

8. Change of variable formula in $\mathbb{R}^n$ for C^1 diffeomorphisms.

References

1. G. de Barra, Measure theory and integration, Wiley Eastern Ltd., 1981.

2. N.L. Carothers, Real Analysis, Cambridge University Press, Indian Edition by Foundation Books, 2006.

3. C.D. Aliprantis and O. Burkinshaw, Principles of Real Analysis, 3rd Edition, Academic Press, 1998.

4. A. W. Knapp, Basic Real Analysis, Birkahuser, Indian Edition by Universities Press, 2010.

5. G.B. Folland, Real Analysis - Modern Techniques and their applications, 2nd Edition, Wiley,1999.

6. I.K. Rana, Measure theory and Integration, 2nd edition, Narosa Publishing, 2000.

Subject Code: MAT074 **Credits: 4**

Probability

1. Counting, Permutations and Combinations Basic principle of counting; Distinguishable and indistinguishable objects; Partitions; Permutations, combinations and distributions; Urn models; Binomial and multinomial coefficients

2. Random Experiments and Probability Sample space; Sample points; Events; Axioms of Probability; Probability of union of events; Sample spaces with equally likely outcomes; Probability as a continuous set function.

3. Conditional Probability and independence of events Motivation for conditional probability; Shrinking of sample space when it is known that a certain event occurred; Conditional probability; Independence of events; independent events and disjoint events; Bayes' Theorem and posterior probabilities.

4. Discrete Random Variables Definition; Distribution; Examples; Probability mass function and distribution function; Properties of a distribution function; Expected value; Function of a random variable and its distribution; Expectation of a function of a random variable; Variance of a random variable; Bernoulli, Binomial, Geometric and negative binomial distributions; Poisson distribution; Hypergeopmetric and Zeta distributions; Distribution functions, means and variances of various distributions mentioned above; Poisson random variable as an approximation of Binomial random variable.

5. Continuous random variables Probability density function and Distribution function; Examples; Expectation and variance of continuous random variables; Need they always exist (Cauchy Distribution)? Uniform distribution; Normal distribution; Use of the table of probabilities of Standard normal distribution; Normal approximation of Binomial distribution; Exponential distribution; Hazard rate functions and their uses; Gamma, Chi-square, Beta and F distributions; Weibull and Cauchy distributions; Distribution of a function of a random variable.

6. Joint distribution of two or more random variables Joint distribution functions; Examples; Covariance between two random variables; Independence of random variables; Uncorrelatedness and independence; pairwise independence and mutual independence; Sums of independent random variables; Marginal and Conditional distributions; Conditional distribution: discrete and continuous cases; Bivariate and multivariate normal distributions; Order statistics; Joint distribution of functions of random variables; Application to linear functions of multivariate normal random variable.

7. More on Moments and Conditional Moments Expectation of sums of random variables; variances and covariances of linear combination of random variables having a joint distribution; Conditional expectation; conditional variance; Moment generating functions; Multivariate normal distribution revisited.

8. Limit Theorems Chebyshev's inequality; Weak law of Large Numbers; Statements of Central Limit Theorem; Strong Law of Large Numbers

References:

1. S. Ross, A first Course in Probability, 6th Edition, Pearson Education, 2006.
2. S. Ross, Probability Models, 8th Edition, Elsevier, 2006.
3. A. Dasgupta, Fundamentals of Probability: A First Course, Springer, 2010.
4. W. Feller, An introduction to Probability Theory and its Applications, Volume 1, 2nd Edition, Wiley, 1969.
5. University of Hawaii at Hilo. Online course on Elementary Probability (Math 421)

Semester VIII

Subject Code: MAT081 **Credits: 4**

Functional Analysis

1. Normed Linear spaces: Definitions and lots of examples: (i) Finite dimensional spaces with a variety of norms, especially $\|\|_1$, $\|\_2$ and $\|\|_\infty$; more generally $\|\|_p$, $1 \leq p \leq \infty$. (ii) Their generalization to sequence spaces ℓ^p, $1 \leq p \leq \infty$, and (iii) the function spaces such as the space of bounded functions, continuous functions on a compact space, Continuous functions with $\|\|_p$-norms, (iv) $C^1[a, b]$ with C^1-norm, Lipschitz functions with a Lipschitz norm; (v) $L^p calM$. (vi) Inner product and Hilbert spaces. Completeness and Banach spaces. Quotient spaces.

2. Continuous linear maps: Bounded linear maps. Lots of examples; Any continuous linear map from K^n to any normed linear space is continuous;

3. Equivalent norms as those which induce the same topology and the characterization; All norms on a finite dimensional vector space are equivalent and hence any finite dimensional normed linear space is complete.

   Riesz lemma; a normed linear space is locally compact iff it is finite dimensional.

   Existence of a unique extension of a continuous linear map defined on a dense subset of normed linear space taking values in a Banach space; compare and contrast this with the analogous result in the context of metric spaces.

4. Operator norm of a continuous linear map. Examples of computation of such norms. The norm of a symmetric linear map $\mathbb{R}^n$ with the Euclidean norm is the maximum of absolute values of the eigenvalues of the linear map.

   The algebra of bounded linear operators.

5. Pillars of functional analysis: Hahn-Banach, Uniform boundedness principle, open mapping theorem and closed graph theorem. Applications to some problems of classical analysis.

6. Hilbert spaces: existence of elements of smallest norm in a closed convex subset of a Hilbert space, orthogonal direct sum, orthonormal basis, Bessel's inequality and Parsavel identity. Fourier series.

7. Compact operators: Integral operators, Fredohlm alternative and the spectral theorem for compact operators. Applications to integral equations. Spectral theorem for Compact self-adjoint operators. Application to Sturm-Lioville problem (if time permits).

8. Dual spaces: duals of ℓ^p and $L^p\mathcal{M}$, $1 \leq p < \infty$, dual of $C[a, b]$, dual of a Hilbert space.

9. Weak-* topology and Banach-Alaoglu theorem. Preliminary discussion on product topology, Pointwise convergence of functions, Tychonoff's theorem without proof.

References

1. J. Maddox, Elements of Functional Analysis, 2nd edition, Cambridge University press, 1989.
2. B.V. Limaye, Functional Analysis, 2nd edition, New Age international Ltd., 1996.
3. B. Bollabas, Linear Analysis, an introductory course, CUP, Indian Edition by Foundation Books, 1994.
4. E. Kreyzig, Introductory Functional Analysis with applications, John Wiley & Sons, 2001.
5. Y. Eidelman, V. Milman and A. Tsolomitis, Functional Analysis, An Introduction, Indian edition by Universities Press, 2010.

Subject Code: MAT082 **Credits: 4**

Complex Analysis

1. The field of complex numbers, Polar representation of complex numbers, Euler's identity,de Moivre's theorem. Argument and logarithm of a nonzero complex numbers. Non-existence of a continuous argument on $\mathbb{C}^*$.

   $\mathbb{C}_\infty := \mathbb{C} \cup \{\infty\}$, the extended complex plane as the one point compactification of the locally compact Hausdorff space $\mathbb{C}$. It is homeomorphic to $S^2 \subset \mathbb{R}^3$ via stereographic projection.

2. A very quick review of the metric space structure of $\mathbb{C}$, sequences and series and their convergence, continuity, uniform convergence of a sequence of functions, power series (All of this was done in Analysis – II).

3. Complex differentiability; Cauchy-Riemann equations, Geometric meaning of C-R equations, holomorphic functions as functions differentiable on an open subset of $\mathbb{C}$ versus analytic functions which are locally representable as a power series. Cauchy's theory establishes that these two are the same.

   Cauchy's theorem leads us to the existence of a holomorphic (branch of) logarithm on star-shaped domains, in particular, the complex plane minus a half-ray emanating from 0.

4. Path integrals: Integration of continuous complex valued functions on an interval, fundamental theorems of integral calculus, piece-wise C^1-path, path integrals, independence of path in path integrals, primitives. Any Cauchy type integral on a path $\int_\gamma \frac{f(w)}{w-z}$ for $z \in \mathbb{C} \setminus \gamma$ is an analytic function.

   Winding numbers of paths;

5. Cauchy Theory: Goursat's theorem and Cauchy's theorem for a convex/star-shaped open subsets, Cauchy integral formula, analytic functions are the same as holomorphic functions.

6. Basic results of complex analysis: Liouvill'es theorem, Morera's theorem, Weierstrass theorem on the limit of locally uniformly convergent sequence of holomorphic functions, Maximum modulus, minimum modulus principles, Open mapping theorem, Schwarz lemma.

7. Isolated singularities: classified via the behaviour $\lim_{z\to z_0} f(z)$. Laurent series, characterization of isolated singularities using Laurent series.

8. Residues: definitions, examples. Residue theorem in the case of a star-shaped domain; Computation of real integrals and summation of series using residue theorem.

9. Argument Principle and Rouche's theorem.

10. Conformal mappings: Conformal mappings (holomorphic automorphisms) of the three simply connected domains, $\mathbb{C}$, unit disk and $\mathbb{C}_\infty$, the Riemann sphere.

References

1. H.A. Priestley, Complex analysis, 2nd edition,Oxford University Press, Indian Edition, 2006.
2. J. Bak and D.J. Newmann, Complex analysis, 2nd edition, Springer Indian Edition (SIE), 2009.
3. L. Hahn and B. Epstein, Classical complex analysis, Jones & Barlett Student edition, 2011.
4. T.W. Gamelin, Complex analysis, SIE, 2004.
5. J.B. Conway, Functions of one complex variable, 2nd edition, SISE, Narosa, 1996.
6. L.V. Ahlfors, Complex Analysis, 3rd edition, McGraw Hill International edition, 1979.

Subject Code: MAT083 **Credits: 4**

Graph Theory

1. Basic Concepts : Various kinds of graphs, simple graphs, complete graph, walk, tour, path and cycle, Eulerian Graph, bipartite graph (characterization), Hypercube graph, Petersen graph, trees, forests and spanning subgraphs, distances, radius, diameter, center of a graph, the number of distinct spanning trees in a complete graph.

2. Matchings : augmenting path, Hall's matching theorem, vertex and edge cover, independence number and their connections, Tutte's theorem for the existence of a 1-factor in a graph.

3. Graph Colourings : chromatic number, Greedy algorithm, bounds on chromatic numbers, interval graphs and chordal graphs (with simplicial elimination ordering), Brook's theorem and graphs with no triangles but large chromatic number, chromatic polynomials.

4. Hamilton Property : Necessary conditions, Theorems of Dirac and Ore, Chvatal's Theorem and toughness of a graph, Non-Hamiltonian graphs with large vertex degrees.

5. Planar graphs : Embedding a graph on plane, Euler's formula, non-planarity of K_5 and $K_{3,3}$, classification of regular polytopes, Kuratowski's theorem, (no proof), 5-colour theorem.

6. Ramsey Theory : Bounds on $R(p, q)$, Bounds on $R_k(3)$: colouring with k colours and with no monochromatic K_3, application to Schur's theorem, Erdos and Szekeres theorem on points in general position avoiding a convex m-gon.

References :

1. D.B. West, Introduction to Graph Theory, Prentice Hall of India, 2001.

2. J.A. Bondy and U.S.R. Murty, Graph Theory with applications, Springer Verlag, 2008.

3. R. Diestel, Introduction to Graph Theory, Springer Verlag, 2017.

4. G. Chartrand and P. Zhang, A First Course in Graph Theory, Dover Publications, 2013.

5. F. Harary, Graph Theory, Addison-Wesley, 1969.

Subject Code: MAT084 **Credits: 4**

Partial Differential Equations I

1. **First order PDE:** Definition and examples. Linear, Non-linear PDE. Solution of first order PDEs: Cauchy problem. Method of characteristics, Lagrange's method. Non-linear first order PDEs. Initial value problems.

2. **Second order PDE** Definition and examples. Classification of second order PDEs in to hyperbolic, elliptic and parabolic PDEs. Canonical forms of three types.

3. **Hyperbolic PDEs** Wave equation. One dimensional wave equation and the initial value problem for homogeneous equations. d'Alembert's formula for the solution. Initial and boundary value problems in finite interval for homogeneous equations. Solution by eigenfunction exapnsion and also d'Alembert's formula and the parallelogram rule. Non-homogeneous wave equation and Duhamel's principle. Higher dimensions: First three dimensional wave equation and then the two dimensional wave equation. If time permits

   (a) Huygen's principle

   (b) the effect of lower order terms on the wave equation

   may be explained.

4. **Elliptic PDEs** Introduction of Laplace equation. Separation of variables technique in solving the Laplace equation when the domain has symmetries. Dirichlet and Neumann boundary value problems for smooth bounded domains. Green's identities. Uniqueness of the solution. Maximum principle. Fundamental solution. Green's function. Poisson kernel. Dirichlet problem on Half space and the ball. Harmonic functions. Liouville theorem. Eigenvalues of the Laplacian. Spherical and rectangular domains may be done.

5. **Parabolic PDEs** Heat equation in a bounded domain. Method of eigenfunction expansion for solutions. Weak maximum principle and uniqueness of solution.

References

1. R. McOwen, Partial Differential Equations, Prentice Hall India, 2003.

2. L.C. Evans, Partial Differential Equations, AMS, Second Edition, 2010.

3. T. Myint-U & L. Debnath, Linear Partial Differential Equations for Scientists and Engineers, Fourth Edition, Birkhauser, 2007.

4. I. N.Sneddon, Elements of Partial Differential Equations, McGraw-Hill, 1957.

5. A. Jeffrey, Applied partial differential equations: An introduction, Academic Press, 2002.

6. T. Amaranath, An elementary course in partial differential equations, Narosa Publishing House, 2003.

7. F. John, Partial differential equations, Fourth Edition, Springer-verlag, New York, 1991.

Semester IX

Subject Code: MAT091 **Credits: 4**

Several Variable Calculus

1. Definition of differentiability of functions from Rm to Rnand examples. Directional derivatives. Partial derivatives and total derivative of differentiable functions,Jacobian matrix,Chainrule, Mean value theorem and applications,interchanging order of derivatives.

2. Higher order derivatives, Taylor's theorem for real valued functions. Maxima/minima of real valued differentiable functions, Lagrange multipliers.

3. Inverse mapping theorem, Implicit mapping theorem and applications.

4. Multiple integrals, Properties of integrals, Existence of integrals, iterated integrals,change of variables.

5. Curl, Gradient, divevergence, Laplacian cylindrical and spherical coordinate, line integrals, surfaceintegrals, Theorems of Green, Gauss and Stokes.

References

1. T.M. Apostol, Mathematical Analysis, Second Edition, Addison Wesley, 1974.

2. T.M. Apostol, Calculus Vol.2, Multi-Variable Calculus and Linear Algebra with Applications to Differential Equations and Probability 2nd Edition, John Wiley & Sons, 1969.

3. L.H. Loomis and S. Sternberg, Advanced Calculus, Addison-Wesley Publishing Company, Inc., 1968.

4. C.H. Edwards, Advanced Calculus of Several Variables, Academic Press Inc, New York, 1973.

5. J. Stewart, Multivariable Calculus, Brooks/Cole, USA, 2008.

Electives

Subject Code: MAT01E **Credits: 4**

Operations Research

1. Review of Linear Algebra, Introduction to Convex Analysis and Polyhedral Theory

2. Matrix Algebra Partitioning of matrices and computing inverse by partitioning. Linear Dependence, Bases, Basic solutions. Convex Analysis Convex sets,Convex functions and their properties, Supporting and Separating hyperplanes Polyhedral theory Extreme points, Extreme directions of Polyhedral sets and Representation of Polyhedral sets

3. Linear Programming Standard Linear Programming Problem- Formulation Theory of Simplex Method Basic Feasible solutions, Geometric motivation and Algebra of the simplex method. Two Phase, Big -M and Single Artificial Variable technique. Degeneracy and Cycling. Revised Simplex Method

4. . Duality Theory Formulation of Dual Problem. Duality theorems. Primal Dual Method and Dual Simplex Method. Sensitivity Analysis.

5. Introduction to Dynamic Programming The Stagecoach Model Reliability problem Dynamic Programming with variables taking continuous values Solve Linear Programming using Dynamic Programming.

6. Transportation and Assignment Problems. Definition of the transportation problem and properties of the A matrix. Finding an initial basic feasible solution and Simplex Method for transportation problems. Hungarian Method of Assignment. Transportation problem as Network problem.

7. Maximal ow and Shortest Path problems. Ford Fulkerson Labeling Algorithm . Dijkstra's Algorithm.

References:

1. M.S. Bazaara, J.J. Jarvis and H.D. Sherali, Linear Programming and Network Flows, 2th edition, John Wiley, New York, USA, 1990. Additional References:

2. H. Taha, Operations Research , Pearson Education, 2011.

3. G. Hadley, Linear Programming, , Narosa Publishing House, 1997.

4. C.H. Papadimitriou and K. Steiglitz, Combinatorial Optimization, Dover Publications, 1998.

5. F.S. Hillier and G.J. Lieberman, Introduction to Operations Research , McGraw-Hill, 8th Edition, 2001.

Subject Code: MAT02E **Credits: 3+2**

Computational Mathematics

1. **FINITE DIFFERENCE METHOD**

    1 Parabolic equations: Explicit and Crank-Nicolson Schemes for - weighted average approximation - derivative boundary conditions - Truncation errors - Consistency, Stability and convergence- Lax Equivalence theorem- eigenvalues of a common tridiagonal matrix - Gerischgorin's theorems - stability by matrix and Fourier-series method - A.D.I. method.

    2 Hyperbolic Equations: First order quasi-linear equations and characteristics - numerical integration along a characteristic - Lax-Wendroff explicit method - second order quasi - linear hyperbolic equation - characteristics - solution by the method of characteristics - Explicit method for linear hyperbolic equations.

    3 Elliptic Equations: Solution of Laplace and Poisson equations in a rectangular region using standard five point finite difference formula - five point finite difference formula with non uniform grid - Finite difference in Polar coordinate - Discretisation error - Mixed Boundary value problems.

2. **FINITE ELEMENT METHODS**

    4 Weak formulation of Boundary Value Problems, Ritz-Galerkin approximation, Error Estimates, Piecewise polynomial spaces, Finite Element Method, Relationship to Difference Methods, Local Estimates.

    5 Finite element methods for elliptic problems, error analysis for the finite element method, Galerkin methods for time-dependent problems, error estimates, two-dimensional problems.

3. **Laboratory Assignments (not limited to):**

    (a) Explicit and Crank-Nicholson schemes for with prescribed and derivative boundary conditions

    (b) ADI method for two space dimensional parabolic PDE

    (c) Method of numerical integration along characteristics for first order hyperbolic PDE

    (d) Lax-Wendroff method

    (e) Finite difference method for Laplace and Poisson's equations

    (f) Finite element method for Two point BVP

    (g) Finite element method for one space parabolic PDE

(h) Finite Element method for Poisson's equation.

References

1. G.D. Smith, Numerical Solution of P.D.E., Oxford University Press, New York, 1995.

2. A.R. Mitchel and S.D.F. Griffiths, The Finite Difference Methods in Partial Differential Equations, John Wiley and sons, New York, 1980.

3. K.W. Morton, and D.F. Mayers, Numerical Solutions of Partial Differential Equations, Cambridge University Press, Cambridge, 2002.

4. S. Brenner and R. Scott, The Mathematical Theory of Finite Elements Methods, Springer-Verlag, New York 1994.

5. C. Johnson, Numerical Solutions of Partial Differential Equations by the Finite Element Method, Cambridge University Press, Cambridge 1987.

Subject Code: MAT03E **Credits: 3+2**

Design and Analysis of Algorithms(Theory and Lab)

1. Introduction Algorithms- Analysing algorithms - Designing algorithms- Growth function - recurrences - counting - amortized analysis - advanced data structures - B-trees - hashing - dynamic order statistics - disjoint set data structure

2. Sorting Insertion sort - Divide and Conquer approach -Merge sort - Quick sort - Heap Sort - lower bounds -median and order statistics

3. Graph Algorithms and Dynamic programming Representation of Graphs - Breadth First Search - Depth First Search - Minimum Spanning Algorithms : Prim's and Kruskal's algorithms - Shortest path algorithm: Dijkstra's and Bellman -Ford algorithms, maximum flow problem- dynamic programming - Huffman coding

4. String Matching Naïve string matching algorithm, string matching with finite automata, Knuth -Moris-Pratt algorithm

5. Numerical Algorithms Matrix multiplication : Strassen's algorithm - Representation of Polynomials: DFT, FFT - Number theoretic algorithms - Modular arithmetic, Chinese remainder theorem.

6. Complexity Theory Polynomial time - Complexity classes - class P, NP, NPC - reducibility - NP Completeness problems: Travelling Salesman problem - Hamiltonian cycle problem - Vertex cover - Subset sum problems.

References:

1. T.H. Cormen, C.E. Leiserson, R.L. Rivest, Introduction to Algorithms, Prentice Hall of India, New-Delhi, 2004.

2. S.Basse, Computer Algorithms: Introduction to Design and Analysisng, Addision Wesley, 1993.

3. A. Levitin, , Introduction to the Design and Analysis of Algorithms, Pearson Eduction Pvt. Ltd, New Delhi, 2003.

4. R. Sedgewick and and K. Wayne, Algorithms, Addision Wesley, 2011.

Subject Code: MAT04E **Credits: 4**

Coding Theory

1. Basic Concepts : Idea behind use of codes, block codes and linear codes, repetition codes, nearest neighbor decoding, syndrome decoding, requisite basic ideas in probability, Shannon's Theorem(without Proof)/

2. Good Linear and Non-Linear Codes : Binary Hamming codes, dual of a code, constructing codes by various operations, simplex codes, Hadamard Matrices and codes constructed from Hadamard and conference matrices, plotkin bound and various other bounds, Gilbert-Varshamov bound.

3. Reed-Muller and related codes : First Order Reed-Muller codes, RM Code of order r, Decoding and Encoding using and algebra of finite field with characteristic two.

4. Perfect Codes: Weight enumerators, Kratchouwk Polynomials, Lloyd's theorem, Binary and ternary Golay codes, connections with Steiner systems.

5. Cyclic Codes : The generator and the check polynomial, zeros of a cyclic code, the idempotent generators, BCH codes, Reed-Solomon codes, Quadratic residue codes, generalized RM codes.

6. One of the following topics :

   (a) Codes over Z4 : Quaternary codes over Z4, binary codes derived from such codes, Galois rings over Z4, cyclic codes over Z4

   (b) . Goppa codes, the minimum distance of Goppa codes, generalized BCH codes, decoding of Goppa codes and their asymptotic behavior.

   (c) Algebraic geometry codes, algebraic curves and codes derived from them, Riemann-Roch Theorem.

   (d) Asymptotically good algebraic codes, Justesen Codes.

References:

1. J.H. Van Lint, Introduction to coding Theory, Springer GTM, 1999.

2. W.C. Huffman and V. Pless, Fundamentals of Error Correcting Codes Cambridge University Press, 2003.

3. J. McWilliams and Sloane, Coding Theory, North-Holland, 1983.

Subject Code: MAT05E **Credits: 4**

Mathematical Methods

1. INTEGRAL EQUATION: Introduction: Types of Integral equations, Integral equations with separable kernels - Reduction to a system of algebraic equations, Fredholm alternative, an approximate method, Fredholm integral equations of the first kind, method of successive approximations - Iterative scheme, Volterra integral equation, some results about the resolvent kernel, classical Fredholm theory - Fredholm's method of solution - Fredholm's first, second, third theorems.

2. APPLICATIONS OF INTEGRAL EQUATIONS: Application to ordinary differential equation - Reduction of Initial value problems and boundary value problems to integral equations - Green's function Approach - Singular integral equations - Abel integral equation

3. CALCULUS OF VARIATIONS: Variation of a functional - Euler-Lagrange equation - Necessary and sufficient conditions for extrema - Variational methods for boundary value problems in ordinary and partial differential equations.

4. GENERALIZED FUNCTIONS: Elementary properties - Generalized function as the limit of a sequence of good functions - Differentiation of generalized function - Regularization of divergent integral - Fourier, Laplace Transform of generalized function - Convergence of a sequence of generalized functions. (*If time permits*)

References

1. I.M. Gelfand and G.E.Shilov, Generalized Functions. Volume I: Properties and Operations, Academic Press, 1964.

2. I.M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall, New Jersey, 1963.

2. F.B. Hildebrand, Methods of Applied Mathematics, Dover, New York, 1992.

3. F.G. Tricomi, Integral Equations, Dover Publications, 1985.

4. D. Porter and D. S. G. Stirling, Integral Equations, Cambridge University Press, 1993.

5. L. Elsgolts, Differential Equations and the Calculus of Variations, MIR Publishers, Moscow, 1970.

6. R. Weinstock, Calculus of Variations, with Applications to Physics and Engineering, McGraw-Hill, New York, 1952.

7. R.P. Kanwal, Linear Integral Equations: Theory & Technique, Second Edition, Birkhäuser, 2013.

8. R.P. Kanwal, Generalized Functions: Theory and Applications, Third Edition, Birkhäuser, 2004.

Subject Code: MAT06E **Credits: 4**

Fluid Dynamics

1. KINEMATICS OF FLUIDS IN MOTION
   Real and ideal fluids. Coefficient of viscosity. Steady and unsteady flows. Isotropy. Orthogonal curvilinear coordinates. Velocity of a fluid particle. Material local and convective derivative. Acceleration. Stress. Rate of strain. Vorticity and vortex line. Stress analysis. Relation between stress and rate of strain, Streamline. Path lines. Streak lines. Velocity potential. Eulerian and Lagrangian forms of Equation of continuity. Boundary conditions and boundary surfaces.

2. EQUATIONS OF MOTION OF A FLUID
   Pressure at a point in a fluid. Euler's equations of Motion. Momentum equations in cylindrical and spherical polar coordinates. Conservative field of force. Flows involving axial symmetry. Equations of motion under impulsive forces. Potential theorems.

3. IN VISCID FLOWS
   Energy equation. Cauchy's Integrals. Helmholtz equations. Bernoulli's equation and applications. Lagrange's hydro-dynamical equations. Bernoulli's theorem and applications. Torricelli's theorem. Trajectory of a free jet. Pitot tube. Venturi meter.

4. TWO DIMENSIONAL AND IRROTATIONAL MOTION
   Two-dimensional flows. Stream function. Complex potential. Irrational and incompressible flow, Complex potential for standard two-dimensional flows. Cauchy Riemann equations in polar form. Magnitude of velocity. Sources and sinks in two dimensions. Problems. Kinetic energy of liquid. Theorem of Blasius. Complex potential due to source. Doublet in two dimensions. Milne-Thomson circle theorem. Fflow and circulations. Stoke's theorem. Kelvin circulation theorem. Kinetic energy of infinite liquid. Kelvins minimum energy theorem. Permanence if irrotational motion. Vortex motion. Dynamical similarity. Boundary layer theory.

Reference:

1. G.K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, 1993.

2. F. Chorlton, Text book of Fluid Mechanics , CBS Publishers, New Delhi, 1985.

3. F. White, Viscous Fluid Flow , McGraw -Hill, 1991.

4. M.D. Raisinghania, Fluid Dynamics , S Chand, New Delhi, 2000.

Subject Code: MAT07E **Credits: 4**

Differential Geometry

1. Differential Calculus- A quick review. Inverse mapping theorem (without proof; this has already been done in multivariable calculus and differential geometry).Smooth functions with compact support. Existence and uniqueness theorem in ODE- without proof. Differential manifolds. Smooth maps. Tangent spaces to a manifold. Immersion and submersion. Submanifolds.

2. Covariant Differentiation. Riemannian metric. Local isometry and isometry. Integration. Levi-Civita connection on a Riemannian manifold. Curvature. Parallel transport. Holonomy. Definitions of sectional, Ricci and scalar curvatures. Geodesics. Exponential map. Gauss lemma. Riemannian submanifolds. Totally geodesic submanifolds. Riemannian submersions. Hopf-Rinow theorem. Cartan-Hadamard theorem.

3. Variation of geodesics. Jacobi fields. Conjugate points. Focal points. Index form and Morse Index theorem. Cut points.

4. Rauch and Berger comparison theorems. Synge's theorem. Klingenberg's injectivity radius estimate. Toponogov's theorem and its applications.

References:

1. J. Cheeger and D.J. Ebin, Comparison theorems in Riemannian Geometry, AMS Chelsea publishing, 2008.

2. M.P. doCarmo, Riemannian Geometry, Birkhauser, 1992.

3. P. Petersen, Riemannian Geometry, Graduate Texts in Mathematics, Springer, 2006.

4. S. Gallot, D. Hulin, J. Lafontaine, Riemannian Geometry, Universitext, Springer, 2004.

5. I. Chavel, Riemannian Geometry: A Modern Introduction, Cambridge University Press, 2006.

Subject Code: MAT08E **Credits: 4**

Lie Groups

1. Review of differential Calculus.

2. Differentiable manifolds. Smooth manifolds and diffeomorphisms. Tangent spaces to a manifold. Derivative of smooth maps. Immersions and submersions. Smooth vector fields and their flows. Exponential map. Frobenius theorem.

3. Lie groups and Lie Algebras. Exponential map. Taylor's formula. Cartan's theorem on closed subgroups of a Lie group. Adjoint representation of a Lie group. Lie subgroups and Lie subalgebras. Homogeneous spaces. Abelian Lie groups. Semi simple Lie groups.

References:

1. C. Chevalley, Theory of Lie groups, Princeton University Press, 2000.

2. S. Helgason, Differential geometry, Lie groups and symmetric spaces, Academic Press, 2001.

3. S. Kumaresan, A Course in Differential geometry and Lie groups, TRIM Series, Hindustan Book Agency, 2002.

4. V.S. Varadarajan, Lie groups, Lie algebras and their representations, GTM Series, Springer1984.

Subject Code: MAT09E **Credits: 4**

Advanced Combinatorics

1. Advanced counting numbers : Stirling numbers of the first and second kind and their connections, change of bases for the polynomial vector spaces, Catalan numbers, counting monotone functions, the box office problem and the parking functions.

2. Recurrence Relations: Towers of Hanoi Problem, Solving recurrence relation; homogeneous linear with constant coefficients, repeated roots, difference tables, finding the sums of the first m-th powers and the Bernoulli numbers.

3. Generating Functions : The basic idea; use in solving recurrence relations, Schur's coin exchange problem, exponential g.f. and the e.g.f. of Bell numbers, applications, e.g.f. of Bernoulli numbers and polynomials, Dirichlect generating functions.

4. Partitions : Partitions of integers, Ferrers diagrams, self-conjugate partitions, various identities, Euler pentagonal theorem.

5. Group Actions : Class equation, Sylow's theorems, automorphisms, finite subgroups of the orthogonal group.

6. Polya theory : Group action, Burnside Lemma, Cycle index, weight enumerator, Polya's theorem and applications, de Bruijn generalization.

7. System of distinct representatives : The basic problem, Hall's theorem on SDR, SDR with defect, bipartite graph matchings, interpretation in terms of non-negative matrices, Birkhoff-von Neumann theorem on doubly stochastic matrices.

References

1. S.S. Sane, Combinatorial techniques, Hindustan Book Agency India, 2013.

Subject Code: MAT10E **Credits: 4**

Ordinary Differential Equations II

1. Existence and uniqueness of solutions: Peano existence theorem and Picard's theorem.

2. Oscillation theory and boundary value problems: qualitative properties of solutions, Sturm comparison theorem, Regular Sturm-Liouville problems.

3. Power series solutions, ordinary points, regular singular points, Gauss hypergeometric equation.

   Special Functions: Legendre, Hermite, Chebychev polynomials, Bessel functions.

4. Systems of first order equations: Linear systems, homogeneous linear systems with constant coefficients, nonlinear systems.

5. Nonlinear equations: Types of critical points, stability for linear systems, Lyapunov's direct method, simple critical points of nonlinear systems. Periodic solutions (Poincar'e- Bendixson theorem).

References

1. G.F. Simmons, Differential Equations with Application and Historical Notes, Second Editiion, McGraw Hill Education, 2015.

2. T. Myint-U, Ordinary Differential Equations, Elsevier, North-Holland, 1978.

3. E.A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, Tata McGraw Hill, 1972.

4. M. Braun, Differential Equations and their applications, Fourth Edition, Springer, 1993.

5. S.L. Ross, Differential Equation, Fourth Edition, John Wiley & Sons, 1984.

6. A.K. Nandakumaran, P.S. Datti and R.K. George, Ordinary Differential Equations: Principles and Applications, Cambridge University Press, 2017.

Subject Code: MAT11E **Credits: 4**

Partial Differential Equations II

1. **Nonlinear first-order PDEs:** complete integral, new solutions from envelopes; characteristics;

2. **Introduction to Hamilton-Jacobi equations**: calculus of variations: First variation, Euler-Lagrange Equation, second variation, Hamilton ODE, Legendre transform, Hopf-Lax formula, weak solutions, uniqueness;

3. **Introduction to Conservation laws**: shocks, entropy condition, Lax-Oleinik formula, weak solutions, uniqueness, Riemann's problem, long time behaviour.

4. **Representation of solutions**: separation of variables; similarity of solutions; transform methods (Fourier, Laplace);

5. Converting nonlinear PDE into ODE (Hopf-Cole transform, Asymptotics; Power series (noncharacteristic surfaces, real analytic functions, Cauchy-Kovalevskaya theorem).

References

1. L.C. Evans, Partial Differential Equations, AMS, Second Edition, 2010.

2. P. Prasad & R. Ravindran, Partial Differential Equations, Seceond Edition, New Age International, New Delhi, 2011.

3. F. John, Partial differential equations, Fourth Edition, Springer-verlag, New York, 1991.

4. R.M. Owen, Partial Differential Equations: Methods and Applications, Pearson Education, 2002.

5. J. David Logan, An Introduction to Nonlinear Partial Differential Equations, Second Edition, Wiley-Interscience, New Jersey, 2008.

Subject Code: MAT12E **Credits: 4**

Measure Theoretic Probability

1. Distribution Function Monotone functions; Distribution functions; Absolutely continuous and singular distributions.

2. Probability Measures Classes of sets - field, monotone class, Borel field etc.; Probability measures and their distribution functions

3. Random Variable: Expectation and Independence General definitions; Properties of Mathematical Expectation; Independence

4. Convergence Concepts Modes of convergence; Almost sure convergence; Borel-Cantelli lemma; vague convergence; Continuation; Uniform integrability; convergence of moments.

5. Laws of Large Numbers Simple limit theorems; Weak law of large numbers; Convergence of series; Strong law of large numbers; Applications.

6. Characteristic Function General properties; Convolutions; Uniqueness and inversion; Convergence Theorems; Applications; Representation theorems; Multidimensional case - Laplace transforms

7. Central Limit Theorems Liapounov's theorem; Linderberg-Levy and Linderberg-Feller theorems; Ramifications; Error estimation; Law of the iterated logarithm; Infinite divisibility

8. Random Walk Zero-or-one laws; Basic notions; Recurrence; Fine structure; Continuation

9. Conditioning, Markov Property and Martingale Conditional expectation; Conditional independence; Markov Property; Basic properties of martingales; Inequalities and convergence; Applications

References:

1. K.L. Chung, , A course in Probability Theory, 3rd Edition, Academic Press,2001.

2. K.R. Parthasarathy, Introduction to Probability and Measure, Hindustan Book Agency, 2005.

3. P. Billingsley, Probability and Measure, 3rd Edition, Wiley,1995.

4. S.M. Ross and E.A. Pekoz, A Second course in Probability, www. ProbabilityBookstore.com, 2007.

Subject Code: MAT13E **Credits:3**

Transformation Groups

1. Revision of Group Theory.
2. Isometries in $\mathbb{R}^2$.
3. Affine transformations and projective transformations.
4. Symmetries of Differential Equation.

References

1. S.V. Duzhin and B.D. Chebotarevsky, Transformation Groups for beginners, AMS, 2004.

Subject Code: MAT14E **Credits: 4**

Logic

1. Introduction: What is Mathematics? The languages of Mathematics and its properties; Quantifies ($\forall$ and $\exists$), Lots of examples. Mathematical Truth
2. Domain and subdomains of quantifies, Negation and its quantification, Proof by contradiction, Proof by induction.
3. Set theory: Sets, operations on sets, arbitary union and intersections, cartesian product, Cantor set equivalence, Russell paradox.
4. Relations, equivanlence relations, partial order relations, totally ordered, maximal and minimal, Zorn's lemma (atleast the statement), functions, 1-1 and onto functions, countability, uncountability.
5. The integers, Arithmetic and order properties of the the integers, equivalence of well ordering and induction principle.

References

1. K. Devlin, The Joy of Sets, Undergraduate Text in Mathematics, Second Edition, Springer-Verag, 1993.
2. G.H. Hardy, E.M. Wright, R.H. Brown, J. Silverman, A. Wiles, An Introduction to the Theory of Numbers, Sixth Edition, Oxford University Press, 2008.
3. K.H. Rosen, Discrete Mathematics and its applications, Seventh Edition, Mc-Graw Hill Companies, Inc. 2012.

Subject Code: MAT15E **Credits: 5**

Design & Analysis of Algorithms - 1

1. Introduction to Algorithms, lots of examples, Recurrent relations and closed form solution, Tools and techniques for summation, Manipulation of sum, floor and ceiling functions, Finite and infinite calculus, Problem solving using the tools.

2. Number theory an applied perspective, Divisibility, Introduction to relations and functions, Mod and congruence relation, Application of congruence, Independent Residues.

3. Permutation, Permutation of Multi sets, Combination, Application of Permutation and combination, Combinatorial properties of permutations.

4. Design and analysis of algorithms with examples like Euclid algorithm etc..,

5. Sorting - Insertion sort - Divide and Conquer approach -Merge sort - Quick sort. Asymptotics and analysis. Complexity Theory. Polynomial time - Complexity classes - class P, NP, NPC - reducibility - NP Completeness problems.

6. Scientific computing with open source R.

References

1. T.H. Cormen, , C.E. Leiserson, , R.L. Rivest, Introduction to Algorithms, Prentice Hall of India, New-Delhi, 2004.

2. S. Basse, Computer Algorithms: Introduction to Design and Analysisng, Addision Wesley, 1993.

3. A. Levitin, Introduction to the Design and Analysis of Algorithms, Pearson Eduction Pvt. Ltd, New Delhi, 2003.

4. S. Sedgewick, Algorithms, Addison Wesley, 2011.

Subject Code: MAT16E **Credits: 4**

Fractional Differential Equations

1. Special functions of Fractional Calculus: Gamma Function, Mittag-Leffler Function, Wright Function.

2. Origin and brief history of the Fractional Calculus, Grünwald-Letnikov fractional derivative, Riemann- Liouville fractional derivative, Caputo's fractional derivative.

3. Geometric and physical interpretation of fractional integration and fractional differentiation, Sequential fractional derivatives, Left and right fractional derivatives, Properties of fractional derivatives, Laplace Transforms of Fractional Derivatives.

4. Introduction to Fractional Differential Equations(FDEs), Existence and Uniqueness results for FDEs with R-L derivative, Basic theory of FDEs with Caputo derivative.

5. Methods of solving FDEs : Method of reduction to Volterra integral equations, Laplace transform method for FDEs with R-L derivative and FDEs with Caputo derivative.

References

1. I. Podlubny, Fractinal Differential Equations, Academic Press, San Diego, 1999.

2. A.A. Kilbas, H.M. Srivastava and J.J. Trujillo, Theory and Applications of Fractional Differential Equations, Elsevier, Amsterdam, 2006.

3. K.S. Miller and B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley & Sons, INC, New York, 1993.

4. K. Diethelm, The Analysis of Fractional Differential Equations, Springer, Heidelberg, 2010.

Subject Code: MAT17E **Credits: 3**

Basic Coding Theory

1. Review of Vector Spaces, Rings, Ideals, Finite Fields (2 Lectures).
2. Error Correction , Error Detection, Sphere Packing Bound, Binary Linear Codes, Parity Check Matrices, Minimum Distance, Hamming Codes.
3. Cyclic Codes, Generator and Parity Check Polynomials, BCH and RS codes.
4. Dual Codes, Group of a Code, Reed Solomon Codes.

References:

1. W.C. Huffman and V. Pless, Fundamentals of Error-Correcting Codes, Cambridge University Press, New York, 2003.
2. T. Sengadir, Discrete Mathematics, Pearson Education India, 2009.

Subject Code: MAT18E **Credits: 4**

Game theory

1. Linear algebra: vectors, scalar product, matrices, linear inequalities, solution of linear equations, real vector spaces of finite dimensions, linear transformations.

2. Convex sets and polytopes, convex cones, extreme vectors and extreme solutions for linear inequalities.

3. Linear programming: Example problems, formulation of linear programming problem, primal and dual problem; simplex method and its variations for solving linear programming problems, duality theorem.

4. Two-person games: Examples, definitions and elementary theory; solutions of games, pure and mixed strategies, value of the game and optimal strategies; saddle point and minimax theorem; symmetric games; proof of fundamental theorem of games.

5. Solutions to matrix games: Relation between matrix games and linear programming; solving games by the simplex method; optimal strategies and solutions.

References:

1. D. Gale, The Theory of Linear Economic Models, McGraw-Hill Book Company, London, 1990.

2. V. Chvatal, Linear Programming, Series of Books in the Mathematical Sciences, W.H. Freeman and Company, 1983.

Subject Code: MAT19E **Credits: 3**

Number Systems

1. Axioms of set theory, Russel's paradox, Foundation axiom, ordered pair, relation, function, Peano's postulates and natural numbers, definition and properties of addition on N, definition and properties of multiplication on N, order relation on N and its properties. Equivalence of principle of induction and well ordering property on N, finite and infinite sets, pigeon–hole principle, characterization of finite sets, Schorder-Bernstein theorem, countable sets and their properties.

2. Definition of integers as equivalence classes of pairs of natural numbers, addition, multiplication, order relation, subtraction and their properties on Z, proof of the fact that Z is an integral domain, definition of rational numbers , operations on rational numbers and their properties, Q is an ordered field satisfying Archimedian property, denseness of Q in itself, proof of Q is not having least upper bound property, absolute value on Q.

3. Dedekind's construction of real numbers through cuts, addition, multiplication, and order relation on R, R is an ordered Archimedian field with least upper bound property, Cantor's construction of real numbers through equivalence classes of Cauchy sequences of rational numbers, addition, multiplication, field structure, order relation, completeness of R, least upper bound property of R in Cantor's construction of real numbers, uniqueness of real number system, decimal expansion of a real number, when do two different decimal expansion represent a same real number? When does a decimal expansion represent a rational number?

4. Uncountable set, properties of uncountable sets, R is equinumerous with ever interval having at least two points, R is equinumerous with Rn, definition of cardinality, arithmetic on cardinalities, Aleph naught, aleph and their arithmetic, [ordinals, equivalence of axiom of choice (if time permits)]

References:

1. A. G. Hamilton, Numbers, Sets and Axioms: The Apparatus of Mathematics, Cambridge University Press, Cambridge, 1983.

2. E. Hewitt and K. Stromberg, Real and Abstract Analysis, Springer, New York , 1975.

3. E. Kamke, Theory of Sets, Dover Publications Inc., New York, 1950.

4. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill Inc., New York, 1976.

Subject Code: MAT20E **Credits: 4**

Nonlinear Programming

1. Introduction to Optimization problems.(real life examples, constrained and unconstrained, convex and non-convex etc.,)
2. Convex sets, convex hull, Caratheodory's theorem, Seperation theorem and Farka's lemma. (Standard fixed point theorems without proof after teaching Farka's lemma)
3. Convex functions, first and second derivative convexity characterizations, Euclidean(metric) projection on a convex set.
4. Necessary and sufficient conditions for local and global optimality of a feasible point, Weierstrass Theorem.
5. Definition of descent direction and a sufficient condition for descent direction.
6. Optimality conditions: Definitions of normal cone, cone of feasible directions and tangent cone. Relationship between these cones. Optimality conditions based on these cones.
7. Fritz John optimality conditions and KKT optimality conditions.
8. Different constraint qualifications(Abadie's CQ,Mangasarian-Fromovitz CQ, Slater CQ, Linear independence CQ) and their relationship with KKT optimality conditions.
9. Lagrangian Duality: Lagrangian dual problem, Examples to find the dual of a linear as well as nonlinear programming problems, Lagrange multipliers and its relation to global optimality. Convexity of dual problem.
10. Duality gap and existence of Lagrange multipliers, Global optimality conditions in the absence of duality gap. Saddle point and global optimality.
11. Weak and strong duality theorems for convex programs. Explained how these theorems work for linear and quadratic programming problems.
12. Definition of sub-gradient for a convex function. Example of a dual problem with nondifferentiable objective.
13. Sub-gradient projection algorithm for convex problems.
14. Algorithms and algorithmic maps. Examples of algorithms and algorithmic maps. Zangwill's convergence theorem. (without proof)

References:

1. O. Mangasarian, Nonlinear programming, McGraw-Hill Inc.,US, 1969.

2. M.S. Bazaraa, H.D. Sherali and C.M. Shetty, Nonlinear programming, Wiley-Blackwell, 2006

3. N. Andreasson, A. Evgrafov and M. Patriksson, An Introduction to Continuous optimization, Springer Student litteratur, 2013.

Subject Code: MAT21E **Credits: 4**

Introduction to Lie Algebras

1. Review of the following: exponential and logarithmic functions of real and complex variables; inverse function theorem; triangularizability, diagonalizability and simultaneous diagonalizability of matrices; Jordan Canonical Form; topology: Hausdorff topology, continuity, compactness and connectedness; Groups: Normal groups, homomorphism between groups, nilpotent and solvable groups; total derivatives and chain rule.

2. Topological Groups; The group $GL(n, \mathbb{R})$; Examples of subgroups of $GL(n, \mathbb{R})$; Polar decomposition in $GL(n, \mathbb{R})$; The orthogonal group; Gram decomposition.

3. Exponential and Logarithm of a matrix; total derivative of the exponential.

4. Linear Lie Groups: One parameter semigroups and subgroups; Lie algebra of a linear Lie group; Linear Lie groups as submanifolds; Campbell-Hausdorff formula.

5. Lie algebras: Definitions and examples; nilpotent and solvable Lie algebras; semi-simple Lie algebras.

References:

1. J. Faraut, Analysis on Lie Groups, Cambridge Studies in Advanced Mathematics – 110, Cambridge University Press, Cambridge, 2008.

2. B. Hall, Lie Groups, Lie Algebras, and Representations, Springer International Publishing, Cham, Switzerland, 2015.

3. A. Baker, Matrix Groups: An Introduction to Lie Group Theory, Springer-Verlag, London, UK, 2002.

4. N.J. Higham, Functions of Matrices, SIAM, Philadelphia, 2008.

Subject Code: MAT22E **Credits: 4**

Distributions and Sobolev Spaces

1. **Distribution Theory:** Test functions and Mollifiers, Distribution, Operation on distributios, Multiplication and division of distribution; convergence of distribution.

2. Derivatives of distributions: Distributional derivative, derivative of locally integrable functions. Convolution of Distributions: Direct product, properties of convolution, fundamental solution of linear differential operators.

3. Fourier Transform, properties of Fourier transform, Tempered Distributions.

4. **Theory of Sobolev spaces:** Motivation, weak derivative, definition and basic properties of Sobolev spaces, approximation, extension theorem

5. Poincare inequality, Sobolev inequalities, compactness theorems, Trace theory.

References

1. L.C. Evans, Partial Differential Equations, Second edition, Springer, AMS, 2010

2. S. Kesavan, Topics in Functional Analysis and Applications, Second edition, New Age International Private Limited, 2015.

3. R.A. Adams and John J.F. Fournier, Sobolev Spaces, Second edition, Elsevier, Amsterdam, 2003.

4. S. Salsa, Partial Differential Equations in Action: From Modelling to Theory, Springer, New York, 2008.

5. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, New York, 2011.

6. I.M. Gelfand and G.E.Shilov, Generalized Functions. Volume I: Properties and Operations, Academic Press, 1964.

Subject Code: MAT23E **Credits: 4**

Mathematical Biology

1. **Discrete Models:** Introduction, simple models: Cell division, An insect population, tumor cell growth, Discrete Delay models, Logistic type models.
2. **Continuous Models:** Introduction: Growth models, delay models, models with age distribution Theory, compartment analysis, modeling of Glucose-Insulin kinetics
3. **Applications of continuous models in population dynamics:** Malthus model, Logistic Growth, Alle Effect, Predator-Prey system, Lotka-Volterra model, competition meodels, Mutualism or symbiosis.
4. **Infectious Diseases:** Simple epidemic models (SI,SIS, SIR, SIRS Models), age-dependent epidemic models
5. **Partial Differential Equations:** Reaction diffusion equations, Chemotaxis, Reaction diffusion, Models for animal dispersal, Pattern formation on growing domains. (if time permits)

References.

1. J.D. Murray, Mathematical Biology I: An Introduction, Third edition, Springer-verlag, Berlin, 2002.
2. J.D. Murray, Mathematical Biology II: Spatial Models and Biomedical Applications, Third edition, Springer-verlag, Berlin, 2003.
3. L. Edelstein-Keshet, Mathematical Models in Biology, SIAM, Philadelphia, 2005.

Subject Code: MAT24E **Credits: 4**

Python for Sciences

1. Introduction to linux commands and Vi Editor. Overview of installing and running Python. Python interpretor and IDLE, one more text editor GEANY. Simple commands to use Python as a calculator. Python 2.x vs Python 3.x. Variables, Statements, Getting input from the user, Functions, Modules, Running Python scripts from a Command Prompt. Strings, Concatenating strings, String representation; repr and str; input vs raw$_{i}nput.StringConversions;Me$

2. Conditionals and Loops, Importing libraries, Assignment, Blocks, if statement, else and elif clauses, Nesting Blocks. While loops, for loops, Iteration, Breaking, else clauses in Loops. Printing and Output formatting. Format specifiers like align, sign, width, precision, type etc.,. File operations. Python shell error handling. Python exceptions: Try and Except function.

3. Various programs related to basic mathematics followed by Bisection Method, Newton Raphson Method, Regula Falsi Method, Trapezoidal Rule for integration, Simpsons 1/3rd rule, Euler's method for ODE, RK method of ODE etc.,

4. Numpy and Scipy. Obtaining Numpy and Scipy libraries. Using Ipython. Numpy basics, Array creation, Printing Arrays, Basic operations, Universal functions, Indexing, Slicing and iterating. Changing shapes, stacking and splitting of arrays. Matplotlib and plotting. Scipy: scipy.special, scipy.integrate, scipy.optimize, scipy.interpolate, scipy.fftpack, scipy.linalg, scipy.stats.

References.

1. Dawson, Michael, Python programming for the absolute beginner, 3rd Edition, Cengage Learning,2010.

2. K Vishnu Namboothiri, Python for Mathematics Students, Version 2.1, March 2013. (https://drive.google.com/open?id=0B27RbnD0q6rgZk43akQ0MmRXNG8).

3. Numpy tutorial - https://www.numpy.org/devdocs/user/quickstart.html

4. Beginner's Guide to matplotlib - https://matplotlib.org/users/beginner.html

5. Scipy tutorial - https://docs.scipy.org/doc/scipy/reference/tutorial/index.html

Subject Code: MAT25E **Credits: 4**

Computational Number Theory

1. Introduction to Computational Number Theory; Introduction to design and Analysis of Algorithms. Complexity theory; Polynomial time - Complexity classes - class P, NP, NPC - reducibility - NP Completeness problems: Sudoku and Travelling Salesman problem; Some funda- mental notations for the design and analysis of algorithms.

2. Algorithms for integer arithmetic: Divisibility, GCD, modular arithmetic, modular expo- nentiation, congruence, Chinese remainder theorem, orders and primitive roots.

3. Diophantine Equations, Fermats last theorem and its connection with modular forms; Euler's proof for the special case n=3. abc Conjecture of Oesterle and Masser; its connection with Fermat - Catalan problem.

4. Computing irregular primes of large size and verifying Fermat's last theorem for the exponent of those primes.

5. Computations in permutation group Sn; relation connecting primes, primitive roots and permutations; Computations of transitive and non-transitive permutations; Computation validating the relation for large transitive permutations.

References:

1. H. Cohen, A course in computational algebraic number theory, Graduate Texts in Mathemat- ics, Volume 138. Springer-Verlag, 1993.

2. J. Buhler, R. Crandall, and R. Sompolski, Irregular primes to one million, Mathematics of Computation 59 (1992), 717-722.

3. Hardy G.H., Edward M. Wright, (Roger Heath-Brown and Joseph Silverman), An Introduc- tion to the Theory of Numbers, Oxford University Press.

4. Wagsta, Samuel S., Jr., The irregular primes to 125000. Mathematics of Computation 32 (1978), no. 142, 583 - 591.

5. Tanner, Jonathan W.; Wagsta, Samuel S., Jr. New congruences for the Bernoulli numbers. Mathematics of Computation 48 (1987), no. 177, 341 - 350.

6. Aulicino, D. J.; Goldfeld, Morris A new relation between primitive roots and permutations. The American Mathematical Monthly, 76, 664 - 666 (1969).

7. V.P. Ramesh, R Thangadurai and R Thatchaayini, A Note on Gauss Theorem, The American Mathematical Monthly, (2019)

Subject Code: MAT26E **Credits: 4**

Elementary Number Theory

1. Set Theory - Concept of sets and notations, union and intersection, index set; relations and functions, equivalence relation, partial order relation; Well ordering, Mathematical Induction and their equivalence.

2. Natural numbers; Axioms on Integers, ring structure; Divisibility, Division algorithm; GCD and LCM; Bezouts identity; irreducible elements and prime elements; Euclid's theorem on the equal- ity of irreducible and prime elements in Z; Fundamental theorem of arithmetic.

3. Linear Diophantine equations; Congruences; arithmetic of congruences; Chinese remainder the- orem.

4. The sequence of primes, number of primes; asymptotic notations - Big-O and small-o; Distribu- tion of primes, prime number theorem.

5. Arithmetic functions, Mobius functions, Euler's totient function, properties of Euler's totient function; Dirichlet multiplication.

References:

1. G. Jones and J. Jones, Elementary Number Theory, SpringerVerlag, London, 1998.

2. D. Burton, Elementary Number Theory, 7th ed. Tata McGraw-Hill, 2012.

3. G. H. Hardy and E. M. Wright, An Introduction to the Theory of Numbers, Clarendon Press, Oxford, 5th edition, 1979.

4. T. M. Apostol, Introduction to Analytic Number Theory, Springer, Berlin, 1976.

5. V. P. Ramesh and R. Gowtham, Asymptotic Notations and its Applications, The Ramanujan Mathematical Society (RMS): Mathematics Newsletter, Vol. 28, 4, 10-16, June-September 2017.

Subject Code: MAT27E **Credits: 4**

Finite Fields and Applications

1. Structure of Finite Fields : Characterization of Finite Fields - Roots of Irreducible Polynomials - Traces, Norms and Bases - Roots of Unity and Cyclotomic Polynomials - Representation of Elements of Finite Fields - Wedderburn's Theorem.

2. Polynomials over Finite Fields : Order of Polynomials and Primitive Polynomials - Irreducible Polynomials - Construction of Irreducible Polynomials - Linearized Polynomials- Binomials and Trinomials.

3. Factorization of Polynomials : Factorization over Small Finite Fields - Factorization over Large Finite fields - Calculation of Roots of Polynomials.

4. Equations over Finite Fields : Elementary Results on the Number of Solutions - Quadratic Forms - Diagonal Equations - The Stepanov-Schmidt method.

5. Applications of finite fields: Coding Theory - Combinatorics.

References:

1. Rudolf Lidl Harald Niederreiter, Finite Fields, Addison-Wesley Publishing Company-1983.

2. Timothy Murphy, Finite Fields.

Subject Code: MAT28E **Credits: 4**

Delay Differential equations

Text

References:

Text

Subject Code: MAT29E **Credits: 4**

Foundations of Geometry

Text

References:

Text

Subject Code: MAT30E **Credits: 4**

Algebraic Number Theory

Text

References:

Text

Subject Code: MAT31E **Credits: 4**

Matrix Group

1. Matrices: Matrix Operations; matrices as linear transformations; The general linear groups; Change of basis via conjugation.

2. All matrix groups are real matrix groups: Complex matrices as real matrices; Quaternionic matrices as complex matrices; Restricting to the general linear groups.

3. The Orthogonal groups: The standard inner product on Kn ; Several characterizations of the orthogonal groups; The special orthogonal groups; Low dimensional orthogonal groups; Orthogonal matrices and isometries; The isometry group of Euclidean space; Symmetry groups.

4. The Topology of matrix groups: Open and closed sets and limits points; Continuity; Path-connected; Compact sets; Definitions and examples of matrix groups.

References:

1. Kristopher Tapp, Matrix groups for Undergraduates, American Mathematical Society, 2011.

2. Gilbert Strang, Introduction to Linear Algebra, Wellesley-Cambridge Press; 5 edition (11 August 2016).

3. Hoffman kunze, Linear Algebra, Prentice Hall India Learning Private Limited; 2 edition (2015)

Subject Code: MATS01 **Credits: 2**

History of Mathematics

1. Development of Euclidean Geometry and Non-Euclidean Geometries.
2. The Stories of π, e and i.
3. Mathematics in Different Cultures (with special emphasize on Indian Astronomy).
4. Study of Kanakkathikaram and Lilavathi .
5. Development of Modern Mathematics.

References:

1. G.G. Joseph, Crest of the peacock, Third Edition, Princeton University Press, Princeton, 2011.