CENTRAL UNIVERSITY OF TAMIL NADU, THIRUVARUR

SCHOOL OF MATHEMATICS AND COMPUTER SCIENCES

M.Sc. STATISTICS AND APPLIED MATHEMATICS

COURSE STRUCTURE (2019-2020)

Semester	Course code	Course title	Credits	Page No.
Semester 1	SAM011	Real Analysis	4	2
	SAM012	Linear Algebra	4	3
	SAM013	Probability and Statistics	4	4
	SAM014	Sampling Theory	4	5
Semester 2	SAM021	Differential Equations	4	6
	SAM022	Functional Analysis	4	7
	SAM023	Numerical Analysis (Theory and lab)	4	8
	SAM024	Stochastic Processes	4	9
	SAM**E	Elective 1	4	14
Semester 3	SAM031	Optimization Techniques	4	10
	SAM032	Statistical Computing with "R" (Theory and Lab)	4	11
	SAM033	Mathematical Modeling in Biology	4	12
	SAM**E	Elective 2	4	14
	SAM**E	Elective 3	4	14
Semester 4	SAM041	Statistical Quality Control	4	13
	SAM**E	Elective 4	4	14
	SAM04P	Project work	8	
		Total credits	**72**	

SAM011 **REAL ANALYSIS** **Credits 4**

Briefly recall: Limit, Continuity and differentiation.

UNIT I: Numerical sequences, convergent sequence, Cauchy sequence, bounded sequence, -properties of convergent series, upper limit and lower limit of real sequences, series of complex numbers, Cauchy criterion for series of numbers, absolute convergence, series of non-negative real numbers, geometric series, the number e, Cauchy product of series, Merten's theorem, rearrangement of series, Riemann's theorem on rearrangement of series.

UNIT II: Definition and existence of the integral, Properties of the integral, Integration and Differentiation, rectifiable curve.

UNIT III: Point-wise convergence, Uniform convergence, Uniform convergence and continuity, Uniform convergence and Integration, Uniform Convergence and differentiation. Equi-continuous families of functions, Weierstrass theorem

UNIT IV: Lebesgue Outer Measure-Measurable Sets-Regularity-Measurable Functions-Borel and Lebesgue Measurability-Abstract Measure-Outer Measure-Extension of a Measure –Completion of a Measure, Measurable functions, simple measurable functions, sequence of measurable functions.

UNIT V: Integrals of simple functions-Integrals of Non Negative Functions-The GeneralIntegral-Fatou's Lemma, Monotone convergence theorem, Dominated convergence theorem, Integration of Series-Riemann and Lebesgue Integrals-Product measure, Fubini's theorem.

References

1. W. Rudin, Principles of Mathematical Analysis, 3rd Edition, McGraw-Hill, 1984.
2. T. Apostol, Mathematical Analysis, 2nd Edtion, Narosa Publishing House,1985.
3. G. de Barra, “Measure Theory and Integration”, New Age International (P) Limited, 1996.
4. H.L. Royden, Real Analysis, McMillan Publication Co. Inc. 3rd Edition, 1988.

SAM012 LINEAR ALGEBRA Credits 4

UNIT I: Vector Spaces, Subspaces, Basis of a vector space – Vector spaces with inner products - Gram-Schmidt orthogonalization.

UNIT II: Linear transformation (LT) – Properties – Matrix of a linear transformation – Matrix ofcomposite transformation – Matrix of an inverse transformation – Change of basis - Orthogonal transformation - Dual space.

UNIT III: Systems of linear equations, homogeneous and non-homogeneous systems of linear equations; Existence and uniqueness of solutions, Matrix notation, row-reductions, Gaussian elimination method, Rank of a matrix, inverse of a square matrix.

UNIT IV: Eigen values and Eigen vectors of an LT – left Eigen vectors, right Eigen vectors,Diagonalizable LT – Lambda matrix, Composition of lambda matrices, Operator polynomial, Cayley-Hamilton theorem and minimal polynomial for an LT – Eigen values of matrix polynomials.

UNIT V: Bilinear forms - Canonical reduction – Sylvester's law of inertia-Definitions of quadraticform - Lagrange's reduction – Kronecker's reduction -Reduction involving the Eigen values of the matrix, Generalized Eigen value problem.

References

1. K.Hoffman, and R. Kunze, *Linear Algebra*, second edition, Prentice-Hall, 1978.
2. Strang, G., Linear Algebra and its applications, 8th Indian reprint Indian edition, Cengage Learning, 2011.
3. S.H. Friedberg, A.J. Inseland L.E. Spence, Linear Algebra, 4th edtion,Prentice-Hall of India, 2003.
4. S. Axler, Linear Algebra Done Right, Springer; 2nd Edition, 1997.

<u>SAM013</u> <u>PROBABILITY AND STATISTICS</u> <u>Credits 4</u>

UNIT I : Introduction - Sample space and events - Axiomatic approach to probability – Conditional Probability and Independence- Law of multiplication - Law of total probability and Bayes‘ Theorem, Discrete and continuous random variables - probability mass function and density function - distribution function - Expectation and variance.

UNIT II: Special Discrete distributions: Binomial, Poisson and Geometric –Special Continuous distributions: Uniform, Normal, Exponential and Weibull - Joint probability distributions - marginal and conditional distributions – Independent random variables, Conditional expectation.

UNIT III: Moments and moment generating functions- Sums of independent random variables - Limit theorems: Markov and Chebyshev inequalities, Law of Large numbers, Central Limit Theorem.

UNIT IV: Estimation using the regression line - Correlation analysis -Limitations, errors, and caveats of using regression and correlation analyses - Multiple regression and correlation analysis - Inferences about population parameters – Modeling techniques. Point estimation, interval estimation - Criteria of a good estimator –Interval estimation of mean, proportion, and variance (single sample and two samples) - Maximum likelihood estimator.

UNIT V: Hypothesis Testing: General concepts - Errors in Hypothesis testing - One-and two-tailed tests - Tests concerning mean, proportion, and variance - Tests for Goodness of fit and independence of attributes. Introduction to design of experiments, Analysis of variance - Completely Randomized Design and Randomized Block Design.

REFERENCES:

1. S. Ross, A first Course in Probability, 6th Edition, Pearson Education, 2006.
2. S. Ghahramani, Fundamentals of Probability with Stochastic Processes, Pearson education, 2012.
3. R. E. Walpole, Probability and Statistics for Engineers and Scientists, Prentice Hall, 2012.
4. R. I. Levin and D. S. Rubin, Statistics for Management, Pearson Education, 2012.
5. V.K. Rohatgi and A.K.M.D. Ehsanes Saleh, An introduction to Probability and Statistics, 2nd Edition, Wiley Eastern Ltd., 2001.

SAM014 **SAMPLING THEORY** **Credits 4**

UNIT I: The role of sampling theory, simple random sampling for mean, proportions and percentages, estimation of sample size. Stratified sampling for mean and proportions, optimum allocation, relative precision of stratified random and simple random sampling, effect of deviations from the optimum allocation, effect of errors in the stratum sizes.

UNIT II: Systematic sampling for linear and circular cases, variance of estimated mean, comparison of systematic sampling with simple random sampling and stratified sampling for population with linear trend.

UNIT III: Procedure of selecting a sample with varying probabilities, estimation of the population mean and variance of the estimated mean, sampling with varying probabilities and without replacement, ordered estimates, the Horvitz-Thompson Estimator, Some IPPS sampling without replacement procedures, Rao Hartley and Cochran's procedure.

UNIT IV: Ratio Estimator, bias and mean squared error of the ratio estimator and its approximation, ratio estimates in stratified sampling, comparison of the ratio estimate and the mean per unit.

UNIT V: Regression estimator, bias and mean squared error of the regression estimator, efficiency of the regression estimator, regression estimates in stratified sampling, comparison of the regression estimate with the ratio estimate and the mean per unit.

References

1. W. C. Cochran, Sampling Techniques, second edition, third edition, Wiley Eastern,1977.
2. P. V. Sukhatme, B. V. Sukhatme, S. Sukhatme and C. Asok, Sampling Theoryof Surveys with Applications, Indian Society of Agricultural Statistics, 1954.
3. A. K. Gupta, and DG. Kabe, Theory of Sample Surveys, World ScientificPublishing Co. Pvt. Ltd., 2011.
4. D. Raj, Sampling Theory, Tata McGraw-Hill, 1968.

SAM021 DIFFERENTIAL EQUATIONS Credits 4

UNIT I: First order linear differential equations-The method of successive approximations, Lipchitz condition, Convergence of successive approximations, Existence and Uniqueness of solutions for first order initial value problem

UNIT II: Second order linear differential equations-General solution of homogeneous equations, Non-homogeneous equations, Wronskian, Method of variation of parameters, Boundary value problems, Green's functions, Sturm-Liouville problems.

UNIT III: Introduction, Cauchy's problem of first order equations, linear equations of the first order, integral surfaces passing through a given curve, surfaces orthogonal to given system, nonlinear partial differential equation for first order, compatible systems of first order equations

UNIT IV: Origin of second order PDE classification of second order semi-linear PDE, Hadamard's definition of well-posedness. Laplace's equation: Boundary value problems, maximum and minimum principles, uniqueness and continuity theorems, Dirichlet problem for a circle, Dirichlet problem for a circular annulus, Neumann problem for a circle, theory of Green's function for Laplace's equation.

UNIT V: Heat equation: Heat conduction problem for an infinite rod, heat conduction in a finite rod, existence and uniqueness of the solution; Duhamel's principle for wave and heat equations.Wave equation: D'Alembert's solution, vibrations of a finite string, existence and uniqueness of solution, Riemann method

REFERENCES:

1. M. Braun, Differential Equations and their applications, Fourth Edition, Springer, 1993
2. E. A.Coddington , An Introduction to Ordinary Differential Equations, Prentice Hall of India Ltd.,2002.
3. G.F. Simmons, Differential Equations with Applications and Historical Notes, Tata McGraw Hill, 2003.
4. F. John, Partial Differential Equations, second edition, Springer-Verlag, 1978
5. L.C. Evans, Partial Differential Equations, AMS, 2010.

<u>SAM022</u> <u>FUNCTIONAL ANALYSIS</u> <u>Credits 4</u>

UNIT I: Normed linear space; Banach spaces and basic properties; Riesz lemma, best approximation property; quotient space, Bounded linear operators and their properties, dual space, adjoint of a bounded linear operator.

UNIT II: Inner product space and projection theorem; Hilbert space, Orthonormal bases; Bessel inequality and Parseval's formula; Riesz-Fischer theorem, Orthogonal complement.

UNIT III: Bounded operators on Hilbert spaces and basic properties; Space of bounded operators and dual space; Riesz representation theorem; Adjoint of operators on a Hilbert space;

UNIT IV: Self-adjoint, Normal and Unitary Operators; Examples of unbounded operators; Convergence of sequence of operators, Finite dimensional Spectral theorem

UNIT V: Hahn-Banach Extension theorem; Uniform boundedness principle; Closed graph theorem and open mapping theorem. Some applications

References

1. B.V. Limaye, Functional Analysis, Second Edition, New Age International, 1996.
2. E. Kreyzig, Introduction to Functional Analysis with Applications, Wiley, 2007.
3. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
4. A.E. Taylor and D.C. Lay, Introduction to Functional Analysis, Second Edition, Wiley, 1980.

SAM023 NUMERICAL ANALYSIS (Theory & Lab) Credits 4

UNIT I: Linear system (Direct methods): Gaussian elimination - Pivoting - LU Decomposition; Vector and Matrix norms - Error Analysis and Condition numbers; Linear system (Iterative methods): Gauss-Jacobi and Gauss-Seidel - Convergence considerations; Eigenvalue problem: Power method - Jacobi for a real symmetric matrix.

UNIT II: Interpolation: Lagrange's interpolation - Errors in Lagrange's interpolation - Newton's divided differences - Newton's finite difference interpolation - Optimal points for interpolation - Piecewise Interpolation: Piecewise linear and piecewise Cubic Spline interpolation

UNIT III: Numerical differentiation: Numerical differentiation based on interpolation, finite differences, method of undetermined coefficients; Numerical integration: Newton Cotes formulae - Gaussian quadrature - Errors in Simpson's rule and Gaussian quadrature -method of undetermined coefficients -quadrature rules for Multiple integrals.

UNIT IV: Ordinary Differential Equations Single-Step methods: Euler's method and Modified Euler's method -Taylor series method - Runge-Kutta method of fourth order - Multistep methods: Adams-Bashforth - Adams - Moulton methods - Stability considerations - Two point BVPs: Finite Difference method - Linear problems with Dirichlet and derivative boundary conditions - Stiff equations - Eigenvalue problems.

UNIT V: Elliptic equations: Five point finite difference formula in rectangular region – truncation error; One-dimensional Parabolic equation: Explicit and Crank-Nicholson schemes; Stability of the above schemes - One-dimensional Hyperbolic equation: Explicit scheme.

References

1. K.E. Atkinson, An Introduction to Numerical Analysis, Wiley, 1989.
2. G.M. Phillips and P.J. Taylor, Theory and Applications of Numerical Analysis, Second Edition, Elsevier, 2006.
3. E. Isaacson, H.B. Keller, Analysis of Numerical Methods, Dover Publication, 1994.
4. A. Quarteroni, F. Saleri and P. Gervasio, Scientific computing with MATLAB and Octave,Springer, 2006.
5. S.D. Conte, C. de Boor, Elementary Numerical Analysis, Third Edition, McGraw-Hill Book Company, 1983.

SAM024 STOCHASTIC PROCESSES Credits 4

UNIT I: Stochastic processes and their classification – Markov chain– Examples (Random walk, Gambler's ruin problem)- classification of states of a Markov chain-Recurrence-Basic limit theorem of Markov chains-Absorption probabilities and criteria for recurrence.

UNIT II: Markov chains continuous in time – General pure birth processes and Poisson process, birth and death processes, finite state continuous time Markov chains.

UNIT III: Branching processes discrete in time – Generating functions relations – Mean and variance – Extinction probabilities – Concept of Age dependent Branching process

UNIT IV: Renewal processes – Definition and examples – key renewal theorem – Study of residual life time process

UNIT V: Stationary process – weakly and strongly stationary process – Moving average and Autoregressive processes and their covariance functions – Brownian Motion process – Joint probabilities for Brownian motion process – Brownian motion as a limit of random walk

References

1. S. Karlin, and H.M. Taylor, A First Course in Stochastic Processes, Academic Press, 1975.
2. J. Medhi, Stochastic Processes, 3rd Edition, New age International, 2009.
3. B.R. Bhat, Stochastic Models: Analysis and Applications, New Age Publications, 2004.
4. P.W. Jones, and P. Smith, Stochastic Processes: An Introduction, Arnold Press, 2001.
5. E. Cinlar, Introduction to Stochastic Processes, Prentice-Hall Inc., 1975.
6. D.R. Cox, and H.D. Miller, Theory of Stochastic Processes, 3rd Edition, Chapman and Hall, 1983.
7. S.M. Ross, Stochastic Process, Wiley, 1983.

SAM031 OPTIMIZATION TECHNIQUES Credits 4

UNIT I: Linear Programming: Convex sets, supporting and separating hyperplanes, standard linear programming problem, basic feasible solutions, simplex algorithm and simplex method, geometry of simplex method. Duality in linear programming, duality theorems,

UNIT II: Dual simplex method with justification, post optimality analysis, sensitivity analysis and parametric linear programming. Integer Linear Programming: Introduction, Gomory Cut Method, Branch and Bound Method.

UNIT III: Network Analysis: Definition and formulation, critical path method, Project Evaluation and Review Technique(PERT), Optimal allocation of resources (men-power) through time schedule.

UNIT IV: Queueing Theory: Introduction, steady state solution of M/M/c/∞/FIFO and M/M/C/N/FIFO with associated distributions of queue length and waiting time.(c=1 as particular case)

UNIT V: Non-linear Programming: Quadratic Programming, Kuhn-Tucker Conditions, Wolfe-Beal-Fletcher algorithms for solving quadratic programming problems.

References

1. G. Hadley, linear Programming, Addison-Wesley, 1961.
2. F.S. Hiller, and G.J. Liberman, Introduction to Operations Research,6th Edition McGraw-Hill Int.Ed., 1995.
3. F.S. Hiller, and G.J. Liberman, Introduction to Mathematical Programming, 2nd Edition, McGraw Hill Int. Ed., 1995.
4. H.A. Taha, Operations Research, 6th Edition, Prentice-Hall India Ltd, 1997.
5. A. Ravindran, D.T. Phillips and J.J. Solberge, Operations Research, 2nd Edition, John-Wiley, 1987.

SAM032 STATISTICAL COMPUTING WITH "R" Credits 4

UNIT I: Introduction to R - A programming language and environment for data analysis and graphics. Syntax of R expressions: Vectors and assignment, vector arithmetic, generating regular sequence, logical vector, character vectors, Index vectors; selecting and modifying subsets of data set, Data objects: Basic data objects, matrices, partition of matrices, arrays, lists, factors and ordered factors, creating and using these objects; Functions- Elementary functions and summary functions, applying functions to subsets of data.

UNIT II: Data frames: The benefits of data frames, creating data frames, combining data frames, Adding new classes of variables to data frames; Data frame attributes. Importing data files: import.data function, read.table function; Exporting data: export.data function, cat, write, and write.table functions; Outputting results - sink function, formatting output - options, and format functions; Exporting graphs - export.graph function.

UNIT III: Random numbers from various distributions like uniform, Normal, gamma, exponential, beta, F, poisson, binomial, weibulletc Graphics in R: creating graphs using plot function, box plot, histogram, line plot, steam and leaf plot, pie chart, bar chart multiple plot layout, plot titles, formatting plot axes. Interactively adding information of plot - Identifying the plotted points, adding trend lines to current scatter plot, adding new data to current plot, adding text and legend.

UNIT IV: Loops and conditional statements: Control Statements; if statement, if elseStatement. Looping statement; for loop, repeat, while loop Developing simple programs in R for data analysis tasks, saving programs, executing stored programs, defining a new binary operator, assignment within function, more advanced examples, object oriented programme. Creating function libraries- library function, attaching and detaching the libraries

UNIT V: Performing data analysis tasks: Reading data with scan function, Exploring data using graphical tools, computing descriptive statistics, one sample tests, two sample tests, Goodness of fit tests, Defining Statistical Models: Introduction for defining models, Generic functions for extracting model information.

References

1. J. M. Chambers, Programming with Data: A guide to S language, Springer, 1998.
2. W. N.Venables and B. D. Ripley, S Programming, Springer, 2000.
3. B. S. Everitt, A handbook of Statistical Analysis using S-Plus, Chapman & Hall, 1994.
4. P.Dalgaard, Statistics and computing: Introductory Statistics with R, Springer, 2002.
5. J. Maindonald and J. Braum, Data Analysis and Graphics Using R: An example-based approach Second Edition, Cambridge Series in Statistical and Probabilistic Mathematics, 2007.

SAM033 MATHEMATICAL MODELING IN BIOLOGY Credits 4

UNITI: Discrete Models: Introduction, simple models: Cell division, An insect population, tumor cell growth, Discrete Delay models, Logistic type models.

UNIIT II: Continuous Models: Introduction: Growth models, delay models, models with age distribution Theory, compartment analysis, modeling of Glucose-Insulin kinetics

UNIT III: Applications of continuous models in population dynamics: Malthus model, Logistic Growth, Alle Effect, Predator-Prey system, Lotka-Volterra model, competition meodels, Mutualism or symbiosis.

UNIT IV: Infectious Diseases: Simple epidemic models (SI,SIS, SIR, SIRS Models), age-dependent epidemic models

UNIT V: Partial Differential Equations: Reaction diffusion equations, Chemotaxis, Reaction diffusion, Models for animal dispersal, Pattern formation on growing domains.

References.

1. J.D. Murray, Mathematical Biology I: An Introduction, Third edition, Springer-verlag, 2002.
2. J.D. Murray, Mathematical Biology II: Spatial Models and Biomedical Applications, Third edition, Springer-verlag, 2003.
3. L. Edelstein-Keshet, Mathematical Models in Biology, SIAM, 2005.

SAM041 STATISTICAL QUALITY CONTROL Credits 4

Unit I: Shewhart Control Charts for X, R, np, p, c etc., and their uses, OC and ARL of Control Charts, Control Charts based on C.V., Modified Control Charts, CUSUM procedures, use of V-mask, Derivation of ARL.

Unit II: Decision Interval Schemes for CUSUM charts. Economic Designs of Control Charts, Pre-control, Relative Precision and Process Capability analysis and Gauge capability analysis, Multivariate Control charts and HotellingT2.

Unit III: Basic Concepts of Acceptance Sampling, Single, Double, Multiple and Sequential Sampling Plans for Attributes, Curtailed and Semi Curtailed Sampling. Dodge-Romig Tables-LTPD and AOQL protection (Single Sampling Plan only). MIL-STD-105D.

Unit IV: Variable Sampling: Assumptions, Single and Double Variable Sampling Plans. Application of Normal and Non-central t-Distributions in Variable Sampling. Continuous Sampling Plans:CSP-1, CSP-2 and CSP-3. Special Purpose Plans: Chain Sampling Plans, Skip-lot Plans.

Unit V: Concept : Hazard Function and Reliability Function. Exponential, Gamma and Weibull Failure Models.Models for wearout failures. System Reliability-Serial, parallel and mixed systems.

References.

1. D.C. Montgomary, Introduction to Quality Control, John Wiley, 1985.
2. E.G. Schilling, Acceptance Sampling in Quality Control, Marcel Dekker, 1982.
3. I.W. Burr, Statistical Quality Control Methods, Marcel Dekker, 1976.
4. H.J.Mittag and H.Rinne, Statistical Methods of Quality Assurance, Germany Chapman & Hall India, 1993.
5. G.B. Whetherill, Sampling Inspection and Quality Control, Halsted Press, 1977.
6. E.R. Ott,Process Quality Control, McGraw Hill, 1975.
7. S. Halpern, An Introduction to Quality Control and Reliability, Prentice Hall of India, 1979.

List of Electives (*This list may be extended if needed*)

SAM01E **ARTIFICIAL INTELLIGENCE** **Credits 4**

UNIT I: The foundations of AI - The History of AI- Intelligent agents- Agent based system. Searching for solution- Uninformed/Blind search - Informed/ Heuristic search - A* search - Hill-climbing search -Constraint satisfaction problem.

UNIT II: Logics – First order logic, Inference in first order logic, Knowledge representation. The planning problem - Planning with state space search - Partial order search - Planning with proportional logic - Planning and acting in the real world. Adversarial planning.

UNIT III: Uncertainty-Probabilistic reasoning - Semantics of Bayesian network -Approximate inference in Bayesian network, Exact inference in Bayesian network - Probabilistic reasoning over time.

UNIT IV: Learning from observation - Knowledge in learning -Statistical learning methods - Reinforcement learning.

UNIT V: Basics of utility theory, decision theory, sequential decision problems, elementary game theory, sample applications.

References

1. S. Russell and P. Norvig, Artificial Intelligence: A Modern Approach‖, Pearson Education, 2014.
2. D. Pool and A. Mackworth, Artificial Intelligence: Foundations of Computational agents‖, Cambridge University, 2011.

SAM02E **BIO-STATISTICS** **Credits 4**

UNIT I: Functions of survival time, survival distributions and their applications viz. exponential, gamma, Weibull, Rayleigh, lognormal, death density function for a distribution having bath-tub shape hazard function. Tests of goodness of fit for survival distributions.

UNIT II: Analysis of epidemiologic and clinical data: studying association between a disease and a characteristic: (a) types of studies in epidemiology and clinical research (i) prospective study retrospective study (iii) cross-sectional data, (b) dichotomous response and dichotomous risk factor: 2x2 tables (c) expressing relationship between a risk factor and a disease (d) inference for relative risk and odds ratio for 2x2 table, sensitivity, specificity and predictivity. Cox proportional hazard model.

UNIT III: Type I and type II censoring schemes with biological examples, estimation of mean survival time and variance of the estimator for type I and type II censored data with numerical examples. Nonparametric methods for estimating survival function and variance of the estimator.

UNIT IV: Competing risk theory, indices for measurement of probability of death under competing risks and their interrelations. Estimation of probabilities of death under competing risks by maximum likelihood.
Theory of independent and dependent risks.

UNIT V: Simple and general epidemic models (by use of random variable technique). Basic biological concepts in genetics, Mendels law, Hardy-Weinberg equilibrium, random mating, distribution of allele frequency (dominant/co-dominant cases), approach to equilibrium for X-linked genes, natural selection, mutation, genetic drift, equilibrium when both natural selection and mutation are operative, detection and estimation of linkage in heredity.

References

1. D.Collett, Modelling Survival Data in Medical Research, 3rd Edition, CRC Press, 2014.
2. L. M. Friedman, C. D. Furburg, D. L. DeMets,Reboussin and C.B. Granger, Fundamentals of Clinical Trials, 5th Edition, Springer, 2015.
3. A.Indrayan, Medical Biostatistics, 3rd Edition, CRC Press, 2012.
4. E.T. Lee, and J.W. Wang, Statistical Methods for Survival Data Analysis, Wiley, 2013.

SAM03E CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS Credits 4

UNIT I: Inner Product spaces, Norm, Hilbert space, Regularity Conditions, Special kinds of Kernel, Classification of integral equation, Convolution integral, Relation between differential and integral equations, Classification, Conversion of Volterra Equation to ODE, Conversion of IVP and BVP to Integral Equation

UNIT II: Fredholm integral equations, Solution of Fredhlom integral equation using decomposition method, direct computation, Adomain decomposition, successive approximation and successive substitution methods.

UNIT III: Volterra Integral equations, Solution of Volterra integral equation using successive approximation method, Adomian decomposition method, series solution, successive substitution method, resolvent kernel, Volterra integral equation of first kind, Integral equations with separable kernels.

UNIT IV: Introduction, problem of Brachistochrone, Isoperimetric problem, Variation and its properties, functions and functionals, Variational problems with the fixed boundaries, Euler's equation, Functionals in the form of integrals, special cases containing only some of the variables, Functionals involving more than one dependent variables and their first derivatives, the system of Euler's equations, Functionals depending on the higher derivatives of the dependent variables, Functionals containing several independent variables, Variational problems in parametric form

UNIT V: Variational problems with moving boundaries, one sided variations, variational problems with subsidiary conditions, Isoperimetric problems, Numerical methods for solving varitional problems, Rayleigh – Ritz method, Galerkin's Method.

References

1. M. Gelfand, S. V., Fomin, Calculus of variations. Prentice-Hall, 1963
2. Ram P. Kanwal, Linear Integral Equations Theory and Technique, Birkhauser, 1997
3. L. Elsgolts, Differential Equations and the Calculus of Variations, University Press of the Pacific, 2003.

SAM04E DESIGN AND ANALYSIS OF EXPERIMENTS Credits 4

UNIT I: Principles of Experimental Design: Need for designed experiments, how data are obtained, difference between data obtained from designed experiments and

UNIT II: sampling, randomization, replication and blocking-the need for them and how to achieve these principles in an experiment.

UNIT III: Analysis of different designs, their strengths and when to employ which design, completely randomized design, fixed effects and random effects, randomized block design-without and with interaction;

UNIT IV: Latin Square design, repeated LSs; incomplete block designs, balanced incomplete block design(BIBD);

UNIT V: Analysis of covariance, factorial designs-2*k*factorial experiments, confounding, 2*k*-1 fractional factorial experiments, response surface method to determine optimum factor level combination with data obtained from a 2*k*factorial design; split plot design; partially balanced incomplete block design (PBIBD).

References

1. D. C. Montgomery, *Design and Analysis of Experiments*, fifth edition, John Wiley, 2008.
2. M.N. Das, and N. C. Giri, *Design and Analysis of Experiments*, second edition, Wiley Eastern, 1991.
3. A. Dey, *Incomplete Block Designs*, Hindustan Book Agency, 2010.

<u>SAM05E</u>   <u>ECONOMETRICS</u>   <u>Credits 4</u>

UNIT I: Econometric modelling, data and methodology, simple linear regression model. Multiple linear regression model: Estimates, statistical inference, asymptotic properties, units of measurement, functional forms, predictions and analysis of residuals, analysis with qualitative information dummy variables.

UNIT II: Problems with specification and data, multicollinearity. Generalized linear model: Heteroskedasticity, logistic growth model, estimates of demand.

UNIT III: Basic time series regression analysis, dynamic models, non-stationary, time series and spurious regression, autocorrelation and heteroskedasticity in time series analysis.

UNIT IV: Static and dynamic neoclassical production function. Duration models.

References

1. P. J. Dhrymes, Introductory Econometrics, Springer-Verlag, 1985.
2. W. H. Greene, Econometric Analysis, fifth edition, Pearson Education, 2006.
3. J. Johnston, Econometric Methods, McGraw-Hill, 1991.

SAM06E **FINITE ELEMENT METHODS** **Credits 4**

UNIT I: Weak formulation of Boundary Value Problems, Ritz-Galerkin approximation, Error Estimates, Piecewise polynomial spaces, Finite Element Method, Relationship to Difference Methods, Local Estimates.

UNIT II: Review of Lebesgue integration theory, Weak derivatives, Sobolev norms and associated spaces, Inclusion relations and Sobolev's inequality, Trace Theorems, Negative norms and duality.

UNIT III Review of Hilbert spaces, projections onto subspaces and Riesz representation theorem, Symmetric and nonsymmetricvariational formulation of elliptic and parabolic boundary value problems, Lax-Milgram Theorem, Error estimates for General Finite Approximation, Higher-dimensional examples

UNITIV: The Finite Element, Triangular finite elements, Lagrange element, Hermite element, Rectangular elements, Interpolant, Averaged Taylor polynomials, Error representation, Bounds for the Interpolation error, Inverse estimates

UNIT V: Variational formulation and approximation of Poisson's and Neumann equations, Coercivity of the variational problem, Elliptic regularity estimates, Variational approximations of general Elliptic and Parabolic problems, Negative – Norm estimates.

References

1. S. Brenner andR. Scott, The Mathematical Theory of Finite Element Methods, Springer-Verlag, 1994.
   2. C. Johnson, Numerical Solutions of Partial Differential Equations by the Finite Element Method, Cambridge University Press, 1987.
2. P.G. Ciarlet, The Finite Element Methods for Elliptic Problems, SIAM, 2002.
3. T.V.Galerkin Finite Element Methods for Parabolic Problems, Springer-Verlag, 1984.

SAM07E **FLUID DYNAMICS** **Credits 4**

UNIT I:Continuum hypothesis, forces acting on a uid, stress tensor, analysis of relative motion in the neighborhood of a point, Euler's theorem, equation of continuity, Reynolds transporttheorem, conservation of mass, material surface, momentum equation.

UNIT II: Stream lines, Bernoulli's theorem, energy equation, circulation, Kelvin's circulation theorem,vorticity, Lagrange's theorem on permanence of vorticity, two dimensional irrotationalflow ofan incompressible fluid, Milne-Thomson circle theorem, Blasius' theorem, flow past an airfoil,the Joukowski transformation, theorem of Joukowski and Kutta.

UNIT III: Axisymmetric flows, Stokes stream function, Butler's sphere theorem, flows due to source,doublet, uniform flow past a sphere, irrotational three dimensional flow, Weiss' sphere theorem.

UNIT IV: Constitutive equations for incompressible uids, derivation of Navier-Stokes equations, unidirectional flows, Poiseuilleflow, Couetteflow, Stokes first and second problems, stagnationpoint flows, dynamical similarity and Reynolds number.

UNIT V: Flows at low Reynolds number, axisymmetric flow of a viscous fluid, uniform flow past asphere at low Reynolds number, torque and drag on a sphere due to a uniform flow,Prandtl model for boundary layer, boundary layer equation, solution for a flow past a plate.

References

1. G.K. Batchelor, An Introduction to Fluid Mechanics, Cambridge University Press,1993.
2. Happel, J. and Brenner, H., Low Reynolds Number Hydrodynamics with SpecialApplications to Particulate Media, Prentice-Hall, Inc., Englewood Cli_s, N.J., 1965.
3. L.D. Landau, and E.M. Lifshitz, Fluid Mechanics, Pergamon Press, , 1959.
4. T. Kambe, Elementary Fluid Mechanics,World Scientific Publishing Co. Pvt. Ltd.,2007.
5. M.E .O'Neill,and F. Chorlton, Ideal and Incompressible Fluid Dynamics, John Wiley & Sons, Inc., 1986.
6. A.J. Chorin and J.E. Marsden, A Mathematical Introduction to Fluid Mechanics,third edition, Springer-Verlag, 1993.

SAM08E **INTEGRAL TRANSFORMS** **Credits 4**

UNIT I: The Fourier Transform, Algebraic properties of Fourier transform, Convolution, Translation, Modulation. Analytical properties of Fourier transforms, Transform of derivatives and derivatives of transform, Perseval formula, Inversion theorem, Plancherel's theorem, Application to solving ordinary and partial differential equation.

UNIT II: The Laplace transform: Algebraic properties of Laplace transform, Transform of derivatives and derivatives of transform. The inversion theorem, Evaluation of inverse transforms by residue. Asymptotic expansion of inverse transform, Application to solving P.D.E., Integral equation, etc.

UNIT III: The Z-Transform: Properties of the region convergence of the Z-transform. Inverse Z-transform for discrete-time systems and signals, Signal processing and linear system.

UNIT IV: The Hankel transform: Elementary properties; Inversion theorem; transform of derivatives of functions; Perseval relation; Relation between Fourier and Hankel transform; use of Hankel transform in the solution of PDE.

UNIT V: The Mellin transform: Definition; properties and evaluation of transforms; Convolution theorem for Mellin transforms; applications to integral equations.

References

1. D. Loknath, Integral Transforms and their Application, CRC Press, 1995.
2. R. S. Pathak, Integral Transform for Generalized Functions and their Applications. Gordon and Breach Science Publishers, 1997.
3. F.C. Titchmash, Introduction to the theory of Fourier Integrals, Oxford Press, 1937.
4. E.J. Watson, Laplace Transforms and Application, Van Nostland Reinhold Co. Ltd., 1981.
5. E.I. Jury, Theory and Application of Z-Transform, John Wiley and Sons, 1973.
6. W. Rudin, Real and Complex Analysis, Mc. Graw Hill Inc., 1987.

SAM09E **MACHINE LEARNING** **Credits 4**

UNIT I: Introduction to Machine Learning, Different Forms of Learning, Linear Regression, Ridge Regression, Lasso, Bayesian Regression, Regression with Basis Functions.

UNIT II: Instance-Based Classification**,** Linear Discriminant Analysis, Logistic Regression, Large Margin Classification, Kernel Methods, Support Vector Machines, Multi-class Classification, Classification and Regression Trees.

UNIT III: Neural Networks: Multi-layer Networks, Back-propagation, Multi-class Discrimination, Training Procedures, Localized Network Structure, Deep Learning.
Graphical Models: Hidden Markov Models, Bayesian Networks, Markov Random Fields, Conditional Random Fields.

UNIT IV: Ensemble Methods: Boosting - Adaboost, Gradient Boosting, Bagging - Simple Methods, Random Forest, Clustering: Partitional Clustering - K-Means, K-Medoids, Hierarchical Clustering - Agglomerative, Divisive, Distance Measures, Spectral Clustering.

UNIT V: Dimensionality Reduction: Principal Component Analysis, Independent Component Analysis, Multidimensional Scaling, and Manifold Learning.

References

1. C. Bishop, Pattern Recognition and Machine Learning, Springer - Verlag, 2006.
2. T. Mitchell, Machine Learning, McGraw Hill Education, 2017.
3. R.O. Duda, P.E. Hart and D.G. Stork, Pattern Classification, Wiley-Blackwell; 2nd edition, 2000.
4. T. Hastie, R. Tibshirani and J. Friedman, Elements of Statistical Learning, 2nd Edition, Springer, 2009.

SAM10E **MECHANICS** **Credits 4**

UNIT I: The Mechanical system – Generalized coordinates – Constraints – Virtual work – Energy and Momentum.

UNIT II: Derivation of Lagrange's Equations – Examples – Integrals of the motion.

UNIT III: Hamilton's Principle – Hamilton's Equations – Other variational principles.

UNIT IV: Hamilton Principle Function – Hamilton-Jacobi Equation – Separability.

UNIT V: Differential forms and Generating Functions – Special Transformations – Lagrange and Poisson Brackets

Reference:

1. D.T. Greenwood, Classical Dynamics, Prentice Hall of India, 1985.
2. H. Goldstein, Classical Mechanics, 2nd Edition, Narosa Publishing House, 2001.
3. J.L.Synge and B.A.Griffth, Principles of Mechanics, 3rd Edition, McGraw Hill Book Co., 1970.

SAM11E **MULTIVARIATE ANALYSIS** **Credits 4**

UNIT I: Reviews of Multivariate Distributions, Multiple and Partial Correlation and Regression, Multivariate Normal Distribution, Marginal and Conditional Distributions - Maximum likelihood Estimators of sample Mean and dispersion Matrix.

UNIT II: Distribution of mean vector and Sample Dispersion Matrix - James-Stein Estimator for the Mean Vector, Wishart Distribution and its Properties (without derivation)- Distribution of Total, Partial and Multiple correlation under null case – Maximum likelihood estimators of total, partial and multiple correlation – Test based on total, partial and multiple correlations.

UNIT III: Tests based on Mean Vectors for one and two Multivariate Normal Distributions - Hotelling'sT2 and MahalanobisD2 test statistics with their null and non-null distributions - Related Confidence Regions - Testing and Illustration using likelihood Ratio Criterion.

UNIT IV: Principal Component Analysis, Factor Analysis Underlying Models and Illustrations, Identification Problem, Estimation - Maximum likelihood Method, Centroid Method, Canonical Correlation – Extraction - Properties.

UNIT V: Classification Analysis using Discriminant functions - Clustering techniques Hierarchical Clustering - Agglomerative techniques, Single Linkage Method, Complete average linkage method – Non-hierarchical method – K-Mean concept of multidimensional scaling and correspondence analysis.

References

1. T.W. Anderson, An Introduction to Multivariate Statistical Analysis, Second Edition, Wiley Eastern, 1980.
2. R. A.Johnson and D.W.Wichern, Applied Multivariate Statistical, 5th Edition.
3. M.Jambu and M. O. Lebeaux, Cluster Analysis and Data Analysis, NorthHolland Publishing Company, 1983.
4. A.M. Kshirsagar, Multivariate Analysis, Marcel Decker.1972.
5. D. F. Morrison, Multivariate Statistical Methods, Second Edition, McGraw Hill, 1976.
6. Reucher, Multivariate Analysis, Academic Press.

SAM12E NETWORKS, GAMES AND DECISIONS Credits 4

UNIT I: Scope and definition of network models - Minimal spanning tree algorithm –Shortest-route problem – Maximal-flow Model.

UNIT II: Network representation – Critical path (**CPM**) computations – Construction of the time schedule – Linear programming formulation of **CPM**– **PERT** calculations

UNIT III: Optimal solution of two-person zero-sum games – Mixed strategies – Graphical solution of (2 x n) and (m x 2) games – Solution of m x n games by linear programming.

UNIT IV: Decision making under certainty: analytic hierarchy process (**AHP**) – Decision making under risk – Decision under uncertainty

UNIT V: Scope of the Markovian decision problem – Finite stage dynamic programming model – Infinite stage model – Linear programming solution.

References

1. H. A.Taha, Operations Research: An introduction, 8th Edition, Pearson Education, 2007.
2. F. S. Hillier and G. J. Lieberman, Introduction to Operations Research, 8th Edition, Tata McGraw Hill, 2005.
3. W. L. Winston, Operations Research, 4thEdition,Thomson – Brooks/Cole, 2003.

SAM13E STOCHASTIC DIFFERENTIAL EQUATIONS Credits 4

UNIT I: Probability spaces - Random variables - Stochastic processes – Brownian motion. Ito Integrals - Construction of Ito integrals – Properties – Extension of the Ito integral.

UNIT III: The one-dimensional Ito formula - The multidimensional Ito formula – The Martingale representation theorem.

UNIT III: Examples and some solution methods - an existence and uniqueness result- weak and strong solutions.

UNIT IV: Introduction – The 1-Dimensional Linear Filtering Problem – The Multidimensional Linear Filtering Problems.

UNIT V: The Markov Property – The Strong Markov Property – The Generator of an Itô Diffusion – The Dynkin Formula – The Characteristic Operator.

References

1. B.Oksendal, Stochastic Differential Equations -An Introduction with Applications, Springer, 2003.
2. P.EKloeden and E. Platen,‖ Numerical Solution of Stochastic Differential Equations, Springer, 1999.
3. S.Cyganowski, P.Kloeden and J. Ombach, From elementary Probability to Stochastic Differential Equationswith Maple, Springer, 2002.

SAM14E STOCHASTIC MODELING IN FINANCE Credits 4

UNIT I: Information set (sigma algebra), filtration, stochastic process, Wiener process (Brownian motion). Conditional expectation, conditional density.

UNIT II: Stochastic integral, stochastic differential equation. Ito formula, derivation of Black-Scholes partial differential equation. Martingales, examples of martingales, sub martingales, super martingales.

UNIT III: Girasanov theorem, risk-neutral probabilities, application in derivates pricing. Derivatives pricing on incompletes markets, stochastic volatility.Change of numeraire, forward probability measure, alternative derivation of Black-Scholes formula.

UNIT IV: Poisson process, jump-diffusion process, generalization of Ito formula, application in derivates pricing.

UNIT V: Numerical solutions of stochastic differential equation. (Euler and Milstein approximation). Stopping time, local martingales, semi martingales, Levy processes, applications.

References

1. R. Durrett, Stochastic Calculus - a Practical Introduction, Taylor & Francis, 1996.
2. P. Hunt and J. E. Kennedy, Financial Derivatives in Theory and Practice, John Wiley & Sons, 2004.

SAM15E **TIME SERIES ANALYSIS** **Credits 4**

UNIT I: Exploratory Time Series Analysis: Forecasting trend and seasonalitybased on smoothing. Methods of Exponential and moving average smoothing; Types and implications of interventions; Outliers, additive and innovational outliers, procedure for detecting outliers

UNIT II: Stationary Stochastic models: weak and strong stationarity, Deseasonalising and detrending an observed time series, Auto-covariance, autocorrelation function (ACF), partial autocorrelation function (PACF) and their properties, Conditions for stationarity and invertibility,

UNIT III: Models for Time Series: Time series data, Trend, seasonality, cycles and residuals, Stationary, White noise processes, Autoregressive (AR), Moving Average (MA), Autoregressive and Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) processes, Choice of AR and MA periods

UNIT IV: Spectral analysis and decomposition: Spectral analysis of weakly stationary process, Periodogram and correlogram analysis, Spectral decomposition of weakly AR process and representation as a one-sided MAprocess – necessary and sufficient conditions, implication in prediction problems.

UNIT V: Modeling Seasonal Time Series: seasonal ARIMA models, estimation and forecasting, Fitting ARIMA models with Box-Jenkins procedure, Identification, Estimation, Verification, Test for white noise, Forecasting with ARMA models.

References

1. N. T. Thomopoulos, Applied Forecasting Methods, Prentice Hall, 1980.
2. G. E. P. Box,G. M. JenkinsandG. C.Reinsel, TimeSeriesAnalysis–Forecasting and Control, Pearson Education, 2004.
3. P. J. Brockwell and R. A. Davis, IntroductiontoTime Series and Forecasting, Springer, 2002.
4. D. C. Montgomery and L. A.Johnson, Forecasting and Time Series analysis, McGraw Hill, 1977.