

Syllabus for Ph.D. Statistics

Entrance Examination

Section A: Research Methodology

Research Methodology: Definition, Characteristics, Objectives, Research and Scientific method, Literature review, types of Research, process of scientific research, Research design. Criteria of Good Research, Defining Research Problem and Problem Solving Techniques. Solving Research Problems: concepts, constructs, definition, research questions, objectives and Methodologies - Theoretical and Operational. Intellectual property rights (IPR) and Plagiarism; publications/patents/start-ups and its e-filing procedures, norms and policies. Scientific Conduct: Ethics with respect to science and research, scientific misconducts: falsification, fabrication and plagiarism; Publication ethics, publications misconduct, Databases and metrics: Indexing databases, citation databases, Impact factor of journal, SNIP, SJR, IPP, Cite score, h-index, g-index, i10 index, altmetrics

Section B : Statistics

Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Descriptive statistics, exploratory data analysis: Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case). Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes. Standard discrete and continuous univariate distributions. sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range.

Methods of estimation, properties of estimators, confidence intervals. Tests of hypotheses: most powerful and uniformly most powerful tests, likelihood ratio tests. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests.

Simple nonparametric tests for one and two sample problems, rank correlation and test for independence. Elementary Bayesian inference.

Gauss-Markov models, estimability of parameters, best linear unbiased estimators, confidence intervals, tests for linear hypotheses. Analysis of variance and covariance. Fixed, random and mixed effects models. Simple and multiple linear regression. Elementary regression diagnostics. Logistic regression.

Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms. Inference for parameters, partial and multiple correlation coefficients and related tests. Data reduction techniques: Principal component analysis, Discriminant analysis, Cluster analysis, Canonical correlation.

Simple random sampling, stratified sampling and systematic sampling. Probability proportional to size sampling. Ratio and regression methods. Completely randomized designs, randomized block designs and Latin-square designs. Connectedness and orthogonality of block designs, BIBD. 2^K factorial experiments: confounding and construction. Hazard function and failure rates, censoring and life testing, series and parallel systems.

Linear programming problem, simplex methods, duality. Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.